ON QUASI-AFFINE TRANSFORMS OF SPECTRAL OPERATORS

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Throughout this paper, "an operator" means a bounded linear transformation defined on a fixed separable Hilbert space H.

It is known [6, Lemma 7] that a spectral subnormal operator is necessarily normal. Here we show, among the other things, that if a quasi-affine transform of a hyponormal (subspectral) operator T is spectral, then T is normal (spectral) (see below for definitions). This, in particular, answers a question raised by J. G. Stampfli in [7, Remark to Theorem 4].

Definitions. (1) An operator T is called *spectral* if $T = S + Q$, where S (called the scalar part) is similar to a normal operator, Q is quasi-nilpotent, and $SQ = QS$. Every spectral operator has a resolution of the identity which is the same as that of its scalar part. The decomposition $T = S + Q$ is called the canonical reduction of T [2, page 1939].

(2) The restriction of a normal (spectral) operator to an invariant subspace is called a *subnormal* (*subspectral*) operator; a *cosubnormal* (*cosubspectral*) operator is the adjoint of a subnormal (subspectral) operator.

(3) An operator T is called *hyponormal* (*cohyponormal*) if $T^*T - TT^* \geq 0$ ($T^*T - TT^* \leq 0$).

(4) For an operator T and a closed subset F of the complex plane $\mathbb{C}$, we define

$$X_T(F) = \{x \in H : \text{there exists an analytic function } f_x\colon \mathbb{C} \setminus F \to H \text{ such that } (\lambda - T) f_x(\lambda) \equiv x\} .$$

(5) An operator T is said to be a *quasi-affine transform* of an operator S if there exists a one-to-one operator W such that $WT = SW$ and WH is dense in H.

We need the following two lemmas.

LEMMA 1. *Let* A, B, *and* C *be three operators such that* $AC = CB$. *Let* g *be an* H-*valued function* (*not necessarily analytic*) *defined on a subset* G *of* $\mathbb{C}$ *such that* $(\lambda - B) g(\lambda) \equiv x$ *for some* $x \in H$. *Then* $(\lambda - A) C g(\lambda) \equiv Cx$.

The proof is trivial.

The next lemma plays an important role in this paper; our main results are easy applications of this lemma and some results due to C. R. Putnam [4] and Radjabalipour [5].

LEMMA 2. *Let* T *be a spectral operator with the resolution of the identity* E. *Let* F *be a closed subset of the plane. Let* $x \in H$, *and assume there exists a bounded function* $g\colon \dot{\mathbb{C}} \setminus F \to H$ *such that* $(\lambda - T) g(\lambda) \equiv x$. *Then* $E(F)x = x$.

Proof. We assume without loss of generality that the scalar part of T is normal. Let $T = S + Q$ be the canonical reduction of T. By [1, Theorem 1 (page 208)],

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