THE CAUCHY PROBLEM WITH INCOMPLETE INITIAL DATA IN BANACH SPACES

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1. INTRODUCTION

Throughout this paper, E denotes a complex Banach space, and A represents a closed operator whose domain D(A) is dense in E and whose range is in E. We assume that the resolvent set $\rho(A)$ is not empty, in other words, that for some complex number λ the operator $R(\lambda; A) = (\lambda I - A)^{-1}$ is everywhere defined and bounded.

We study the problem of existence and uniqueness of solutions of the nth-order operational differential equation

$$u^{(n)}(t) = Au(t) \qquad (t \geq 0) \tag{1.1}$$

that satisfy an estimate of the form

$$|u(t)| = O(e^{\omega t}) \qquad \text{as } t \to +\infty \tag{1.2}$$

and the initial conditions

$$u^{(k)}(0) = u_k \in E \qquad (k \in \alpha). \tag{1.3}$$

Here, ω denotes a real number, n is an integer $(n > 1)$, and α is a predetermined subset of the set $\{0, 1, \cdots, n-1\}$. We also study the dependence of the solutions on the incomplete set of initial data (1.3). (By a solution of (1.1) we mean an E-valued function u that has n continuous derivatives and satisfies (1.1) for $t \geq 0$.) This is a generalization of the usual Cauchy problem, where growth conditions of the type (1.2) are absent but where α in (1.3) consists of all the integers $0, 1, \cdots, n-1$, in other words, where each of the values $u^{(k)}(0)$ $(k = 0, 1, \cdots, n-1)$ is preassigned. In order to delineate clearly the results in the present paper, we sketch briefly the available results in the usual case. We say that the problem

$$u^{(n)}(t) = Au(t) \qquad (t \geq 0), \tag{1.4}$$

$$u^{(k)}(0) = u_k \qquad (0 \leq k \leq n-1) \tag{1.5}$$

is *well posed* if solutions of (1.4), (1.5) exist (their initial data $u_0, u_1, \cdots, u_{n-1}$ arbitrarily chosen in a given dense subspace of E) and depend continuously on $u_0, u_1, \cdots, u_{n-1}$. For $n = 1$, the problem (1.4), (1.5) is well posed if and only if A generates a strongly continuous semigroup (see [12, especially Chapter I, Section 2, Theorem 2.8] and [7, Part I, Theorem 4.1]). Generators of strongly continuous semigroups are in turn characterized by the theorem of E. Hille and K. Yosida (see

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