COMPACT INTERTWINING OPERATORS

T. L. Kriete, III, Berrien Moore, III, and Lavon B. Page

Let T_1 and T_2 be bounded operators acting on Hilbert spaces H_1 and H_2, respectively. A bounded operator X from H_1 to H_2 that satisfies the condition $XT_1 = T_2X$ is called an *intertwining operator* for T_1 and T_2. We denote the class of such operators by $\mathscr{I}(T_1, T_2)$. Clearly, with the usual operator norm, $\mathscr{I}(T_1, T_2)$ is a Banach space.

It is of interest to look for compact operators in $\mathscr{I}(T_1, T_2)$. That nonzero compact operators need not exist in $\mathscr{I}(T_1, T_2)$, even when $\mathscr{I}(T_1, T_2)$ is relatively large, is clear if for example we take T_1 and T_2 to be the simple unilateral shift; it is well known that there exist no compact analytic Toeplitz operators different from zero. P. Muhly [7] in effect found conditions that guarantee the existence of nonzero compact operators in $\mathscr{I}(T_1, T_2)$.

An example of the opposite extreme occurs when both T_1 and T_2 are the identity operator on a Hilbert space. In this case, there are so many compact operators in $\mathscr{I}(T_1, T_2)$ that if we let $\mathscr{C}(T_1, T_2)$ denote the subspace of $\mathscr{I}(T_1, T_2)$ consisting of the compact operators, then $\mathscr{C}(T_1, T_2)$ has $\mathscr{I}(T_1, T_2)$ as its second dual (R. Schatten [11]). Another example of this phenomenon is the case where T_1 is a unilateral shift of arbitrary multiplicity on a separable, complex Hilbert space, while $T_2 = T_1^*$. That $\mathscr{I}(T_1, T_2)$ is here the second dual of $\mathscr{C}(T_1, T_2)$ was shown by R. N. Hevener [5] in case T_1 is the simple unilateral shift, and by Page [9] in case T_1 is a shift of higher multiplicity. Hevener's result is a consequence of the work of Z. Nehari [8] on bounded Hankel operators and of P. Hartman [3] on compact Hankel operators.

This paper concerns the second of the two extreme possibilities, that is, the case where

$$\mathscr{C}(T_1, T_2)^{**} \simeq \mathscr{I}(T_1, T_2)\,.$$

We prove that this biduality relation holds if T_1 and T_2 are compressions of the simple unilateral shift to co-invariant subspaces. (See concluding comments.)

Let $\mathscr{T}$ and m denote the unit circle in the complex plane and normalized Lebesgue measure on $[0, 2\pi)$. The spaces $L^p(dm)$ $(1 \leq p \leq \infty)$ will be the standard spaces of appropriate complex functions on $\mathscr{T}$. Usually, $L^p(dm)$ and its Hardy subspace $H^p(dm)$ will be denoted simply by L^p and H^p. By H_0^1 we mean the subspace of functions f in H^1 for which $\int_0^{2\pi} f(e^{it})\,dm(t) = 0$. Following tradition, we use χ to denote the identity function on $\mathscr{T}$, so that the simple unilateral shift on H^2 is given by $U: f \to \chi f$. For a more detailed discussion, see [4] and [6].

Received June 11, 1970.

The authors acknowledge partial support from NSF Grant GP-8981, NSF Grant GP-14784, and a grant from the North Carolina Engineering Foundation.

Michigan Math. J. 18 (1971).