ON THE GROUP OF AUTOMORPHISMS OF A FINITE-DIMENSIONAL TOPOLOGICAL GROUP

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Let G be a locally compact topological group, and let A(G) be the group of all (bicontinuous) automorphisms of G. There is then a natural topology in A(G) under which A(G) is a topological group. However, this group is not necessarily locally compact. In fact, some otherwise rather well-behaved groups G (such as infinite-dimensional tori) fail to have a locally compact A(G). The main purpose of this work is to show that if G is a connected, locally compact, finite-dimensional topological group, then A(G) is locally compact and is, moreover, a Lie group.

The word *group* will always mean a topological group, and the identity element of a group will be denoted by 1.

1. PRELIMINARIES

Here we collect some standard definitions and some more or less well-known facts.

1.1. The group A(G) corresponding to a locally compact group G is topologized as follows: For a compact subset C of G and a neighborhood U of 1 in G, let W[C, U] be the set of all $\theta \in A(G)$ such that $\theta(x)x^{-1}$ and $\theta^{-1}(x)x^{-1}$ lie in U for all $x \in C$. Then, as C runs through all compact subsets of G, and U through all neighborhoods of 1 in G, the sets W[C, U] form a system of basic neighborhoods of 1 in A(G). Under this topology, A(G) is a topological group.

If G is a compact group, then this topology coincides with the so-called compact open topology, and if moreover G is a Lie group, then this is the same as the relative topology on the subspace A(G) of a general linear group $GL(n, \mathbb{R})$ ($n = \dim G$). In general, however, the topology we defined above is stronger than the compact open topology on A(G). We also remark that if G is connected and locally connected, then the compact subsets C in W[C, U] may be assumed to be connected.

1.2. Let G be a connected, locally compact group. Then G is locally the product of a compact group K and a local Lie group L_ℓ^*, with K and L_ℓ^* normalizing each other. That is, there exists a neighborhood U of 1 in G such that

$$U = K \times L_\ell^* \quad \text{and} \quad [K, L_\ell^*] = \{1\},$$

where, for subsets A and B of G, [A, B] denotes the commutator subgroup of A and B. Since G is connected, the relation $G = KL^*$ holds, where L^* is the subgroup of G defined by

$$L^* = \bigcup_n L_\ell^{*n}.$$

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