ε-MAPPINGS AND GENERALIZED MANIFOLDS, II

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All spaces considered in this paper are compact metric spaces. A map $f: X \to Y$ of a space X onto Y is an *ε-map* ($\varepsilon > 0$) provided $\operatorname{diam} f^{-1}(y) < \varepsilon$, for each $y \in Y$. If Π is a class of polyhedra, we say that X is *Π-like* provided for each $\varepsilon > 0$ there exists a polyhedron $P \in \Pi$ and an ε-mapping $f: X \to P$ onto P (P and f depend on ε) (see Definition 1 in [5]). By an *n-manifold* we mean a closed connected triangulable manifold of dimension n. We are interested in Π-like continua, where Π is a class of n-manifolds. The following is our main result.

THEOREM 1. *Let* X *be a* Π-*like* n-*dimensional absolute neighborhood retract, where* Π *is a class of* n-*manifolds. Then* X *is a locally orientable,* n-*dimensional generalized closed manifold over every principal ideal domain* L (n-gcm_L). *If* Π *is a class of orientable* n-*manifolds, then* X *is also orientable.*

For the definitions of these notions, see [6] and [1] (see also [9]).

Theorem 1 was proved in [6] for the case where Π is a class of orientable n-manifolds. The consequences stated there for the orientable case are now established without this restriction.

Theorem 1 follows from Theorem 1 in [6] and the following result.

THEOREM 2. *Let* X *be a* Π-*like,* n-*dimensional absolute neighborhood retract, where* Π *is a class of nonorientable* n-*manifolds* P. *Let* $\tilde{\Pi}$ *denote the class of orientable* n-*manifolds* $\tilde{P}$ *that are the* 2-*fold covering spaces of* P. *Then* X *admits a* 2-*fold covering space* $\tilde{X}$ *that is a* $\tilde{\Pi}$-*like continuum.*

Remark. Recall that every (triangulable) nonorientable n-manifold P has a uniquely determined 2-fold covering space $\tilde{P}$ that is a (triangulable) orientable n-manifold (see for example [7, pp. 271-272]).

To see that Theorem 2 and [6] imply Theorem 1, consider a Π-like, n-dimensional ANR X, where Π is a class of n-manifolds. By a theorem of T. Ganea [3], there exists an $\varepsilon > 0$ such that all ε-maps of X onto an n-manifold are homotopy equivalences. Therefore, there exists a subclass $\Pi_0 \subset \Pi$ each of whose members is of the same homotopy type as X, and X is Π_0-like. Consequently, either all manifolds in Π_0 are orientable, or all are nonorientable. In the first case, X must be an orientable n-gcm_L, by Theorem 1 of [6]. In the second case, we apply Theorem 2 to obtain a 2-fold covering space $\tilde{X}$ of X that is a $\tilde{\Pi}_0$-like continuum. The spaces $\tilde{X}$ and X are locally homeomorphic, and therefore $\tilde{X}$ inherits the local properties of X. By a theorem of K. Borsuk [2], a compact metric space is an n-dimensional ANR if and only if it is n-dimensional and locally contractible. Since the latter properties are local, we may conclude that $\tilde{X}$ is also an n-dimensional ANR. The class $\tilde{\Pi}_0$ consists of orientable n-manifolds, and so Theorem 1 of [6] implies that $\tilde{X}$ is an orientable (and hence locally orientable) n-gcm_L. Since local orientability is a local property, we conclude that X is a locally orientable n-gcm_L.

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