A THEOREM ON THE AUTOMORPHS OF A SKEW-SYMMETRIC MATRIX

Morris Newman

In this note we consider certain groups of integral matrices, and without further qualification all matrices that appear will be assumed to have integral entries. The propositions obtained below remain true when the matrix elements belong to an arbitrary principal ideal ring, with essentially no change in the proofs.

Let I stand for the identity matrix of order t, and J for the $2t \times 2t$ matrix

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

The group of automorphs of J is the multiplicative group Γ of matrices M such that

$$MJM' = J,$$

and it is called the *symplectic group.*

An element M of Γ is said to be *symplectic.* A matrix M_0 satisfying the condition

$$M_0 J M_0' \equiv J \pmod{n},$$

is said to be *symplectic modulo* n.

The following result, proved in [3], was the key theorem in the determination of the structure of certain quotient groups of matrices.

THEOREM 1. *Suppose that* M_0 *is symplectic modulo* n. *Then a symplectic matrix* M *exists such that* $M \equiv M_0 \pmod{n}$.

Suppose that K is a nonsingular skew-symmetric matrix. The K-*symplectic group* Γ_K is defined as the multiplicative group of matrices N such that

$$NKN' = K,$$

and an element N of Γ_K is said to be K-*symplectic.* If N_0 is a matrix such that

$$N_0 K N_0' \equiv K \pmod{n},$$

then N_0 is said to be K-*symplectic* modulo n.

In her doctoral dissertation [1], Sister Kenneth Kolmer proved the following result for the group Γ_K, which corresponds to Theorem 1 for Γ:

THEOREM 2. *Suppose that* n *is an integer such that* $(n, \det K) = 1$, *and that* N_0 *is* K-*symplectic modulo* n. *Then there exists a* K-*symplectic matrix* N *such that* $N \equiv N_0 \pmod{n}$.

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