MAPPING CYLINDER NEIGHBORHOODS

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Let (X, A) be a pair of spaces having two structures each of which induces, in some way, a neighborhood of A which is a mapping cylinder. We shall show in this paper that the two neighborhoods are *homeomorphic*. For example, let S be a differential structure on X which induces on A the structure of a differential submanifold. Then any open tubular neighborhood of A (that is, a realization of the normal bundle of A for some complete Riemannian metric on X by normal disks of sufficiently small radius) is a mapping cylinder neighborhood. There are many examples of pairs (X, A) admitting more than one such differential structure. Alternatively, if A is a full subcomplex of some triangulation T of X, then an open simplical (that is, regular) neighborhood of A in the first barycentric subdivision of T is a mapping cylinder neighborhood.

We recall that the *mapping cylinder* M_f of a map f of a space X onto a space Y is the disjoint union $X \times [0, 1] \cup Y$ with each (x, 1) identified to $f(x) \in Y$. By identifying each $x \in X$ with $(x, 0) \in M_f$, we consider X, Y as closed subsets of M_f. For any set A in a space, b(A), i(A), and Cl A will denote its set-theoretical boundary, interior, and closure, respectively. Let A be a closed subset of a space X. An open set $U \supset A$ of X is called an *open mapping cylinder neighborhood* (MCN) of A if there exists a map f of b(U) onto b(A) and a homeomorphism h of (Cl U) - i(A) onto M_f such that $h| b(U) \cup b(A) = 1$. Our main result can be stated in the following form.

THEOREM 1. *Let* U, V *be* MCN's *for a closed subset* A *of a space* X. *If* b(U) *and* b(V) *are paracompact and locally compact, then there exists a homeomorphism of* V *onto* U *that leaves pointwise fixed a neighborhood of* A.

In particular, we obtain the following corollary.

COROLLARY 1. *Let* U, V *be* MCN's *for a (not necessarily compact) closed subset* A *of a locally compact metric space* X. *Then there exists a homeomorphism of* U *onto* V *that leaves pointwise fixed a neighborhood of* A.

If A is any subcomplex of a locally finite complex X, then by the *open regular neighborhood* of A, we shall mean the simplicial neighborhood of A in the second barycentric subdivision. Here we use the term complex both for the complex itself and for the underlying topological space.

COROLLARY 2. *Let* T_1, T_2 *be two locally finite triangulations of a closed pair* (X, A). *Let* R_i *denote the open regular neighborhoods of* A *under* T_i. *Then there exists a homeomorphism of* R_1 *onto* R_2 *that leaves pointwise fixed a neighborhood of* A.

It is known [2] that the tangent spaces of a manifold M corresponding to two differentiable structures may not be equivalent as bundles over M. However, there is the following result in which M is considered as embedded in the tangent space as the zero cross-section.

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