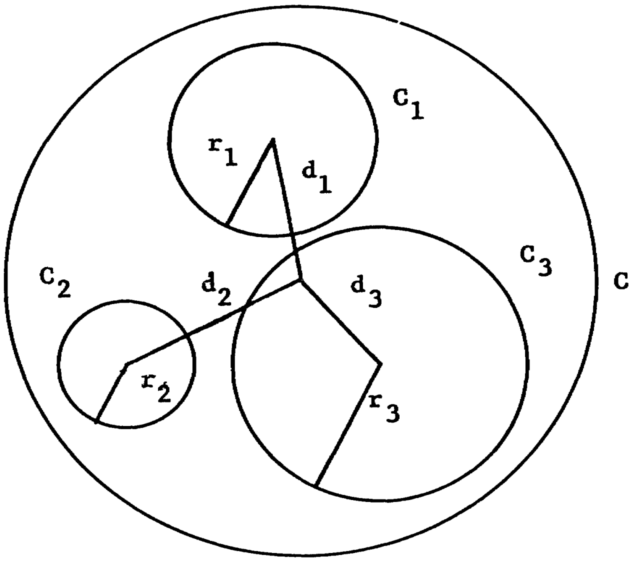
PACKING INEQUALITIES FOR CIRCLES

Philip J. Davis

1. INTRODUCTION

Let the three non-overlapping discs C_1, C_2, C_3 lie inside the unit disc C: $|z| \leq 1$. Let r_i $(i = 1, 2, 3)$ designate the radius of C_i, and let d_i designate the distance from the center of C_i to the origin. Then

$$d_1 d_2 d_3 + r_1^2 + r_2^2 + r_3^2 \leq 1. \tag{1}$$



In this paper, we shall prove (1) and similar inequalities for nonoverlapping discs C_i contained in the unit disc C. Let C_i designate the open disc

$$(x - x_i)^2 + (y - y_i)^2 < r_i^2 \qquad (i = 1, 2, \cdots, n).$$

Our goal is to find simple inequalities relating the quantities x_i, y_i, r_i. As for example, from geometry, we see that a necessary and sufficient condition that C_i and C_j do not overlap is that $(x_i - x_j)^2 + (y_i - y_j)^2 \geq (r_i + r_j)^2$.

2. INEQUALITIES DERIVED FROM REAL VARIABLE THEORY

Note that if $f(x, y)$ is a non-negative integrable function defined on C, then

$$\sum_{i=1}^{n} \iint_{C_i} f(x, y)\, dx\, dy \leq \iint_C f(x, y)\, dx\, dy. \tag{2}$$

Received September 5, 1962.