A REMARK ON SOME ALMOST PERIODIC COMPACTIFICATIONS

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1. The author is indebted to K. de Leeuw for raising the following, at first glance rather bizarre, question: if G is a noncompact, locally compact abelian group, with almost periodic compactification G^*, *is* G^* *the Stone-Čech compactification* $\beta(G^* \setminus G)$ *of* $G^* \setminus G$? (As usual, we view G as a subset of G^*.) At least when the character group $G^\wedge$ of G is not totally disconnected, the answer is *affirmative* (when $G^\wedge$ is totally disconnected, our approach simply fails).

Actually de Leeuw's question is not at all unnatural, since G forms a rather small part, and thus $G^* \setminus G$ a rather large part, of the "large" space G^*, as is more or less well known. For example, Borel subsets of G, that is, elements of the σ-ring generated by compact sets, are of G^*-Haar measure zero, so that, if G is σ-compact, G itself is of G^*-Haar measure zero; a special proof for $G = R$ appears in [2, Thm.4.3], but one can argue that if a Borel set E of G (automatically a Borel set in G^*) is of positive G^*-Haar measure, then $E - E$ has interior in G^*, so that G is imbedded homeomorphically in G^*. As a dense locally compact subgroup, G must fill out all of G^*, and $G = G^*$ is both compact and noncompact.

Since there are few tools available for showing a compact space to be the Stone-Čech compactification of a given subspace, there is probably no need to apologize for our use of the known structure of locally compact abelian groups; and while the result may be classed as a curiosity, it seems worth recording.

The notation used below is standard, as in [3], [4]; however we shall speak of the "direct product," where Kaplansky [3] uses "complete direct sum," for topological suggestiveness (if H is a compact group and we express $H^\wedge$ as a (weak) direct sum, there is a dual representation of H as a direct product, *topologically* and algebraically). Finally, we shall let G^d represent the discretized version of G, so that $G^* = G^{\wedge d \wedge}$ [4, p. 170].

2. Let $\{X_\nu\}$ be an uncountable set of compact Hausdorff spaces, $b = \{b_\nu\}$ an element of the topological product $X = \Pi X_\nu$, and X^b the subspace of X formed by all elements $x = \{x_\nu\}$ with $x_\nu \neq b_\nu$ for at most countably many ν. Then [1, Thm. 2] $\beta(X^b) = X$; therefore clearly $X^b \subset Y \subset X$ implies $\beta(Y) = X$.

Now suppose that G is our noncompact but locally compact abelian group, and that we can represent G^* as a direct product of uncountably many compact groups. (Such a representation is not always possible if $G^\wedge$ is totally disconnected; for example, if $G^\wedge$ is the compact group of p-adic integers, an algebraically indecomposible group, $G^* = G^{\wedge d \wedge}$ is not a product.) Then it will suffice to show that

$$(2.1) \qquad G^* \neq G + G^{*o}$$

where G^{*o} is the subgroup of the product G^* consisting of all elements with at most countably many coordinates different from 0. For then $G^* \setminus G$ contains a coset of G^{*o}, and the result of [1] cited above applies.

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