FABER SERIES AND THE LAURENT DECOMPOSITION

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1. INTRODUCTION

This paper deals with the problem of transfer described by J. L. Ullman [5]. Roughly speaking, we should like to have a method for deciding what statements about a Faber series

$$(1) \qquad \sum_{0}^{\infty} a_n F_n(z)$$

are equivalent to analogous statements about the associated power series

$$(2) \qquad \sum_{0}^{\infty} a_n z^n$$

with the same coefficients.

Ullman partially solved the problem by means of a lemma concerning the rationality of functions: if one of the series (1) and (2) represents a rational function, then the same is true of the other. The lemma leads to an immediate proof, for example, of Iliev's analogue [2] of Szëgo's theorem on power series whose coefficients assume only a finite number of different values.

We shall establish a result (Theorem 2) which is at the same time more elementary and more general than Ullman's lemma. It asserts that if $f(z)$ denotes the mapping function, normalized at $z = \infty$, which is associated with the analytic curve C giving rise to the Faber sequence $\{F_n(z)\}$, then the difference between the series (1) and the series

$$(3) \qquad \sum_{0}^{\infty} a_n [f(z)]^n$$

can be continued so as to be holomorphic everywhere on C and outside of C. The proof of this theorem is based on a very simple tool, the *Laurent decomposition.* This device (developed by the author in connection with the extension of the Faber theory to Riemann surfaces; see [3], [4]) is described in Section 2. In Section 3, we give a brief development of the Faber theory, and we prove our fundamental result. The final section is devoted to applications of the theorem.

Received January 24, 1957.