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Estimates for Nonlinear Harmonic "Measures" on Trees

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1. Introduction

This paper concerns the asymptotic behavior of nonlinear analogs of harmonic "functions" on trees. Our study was motivated by some open problems for p-harmonic functions on domains in $\mathbf{R}^n$. We hope that our results will suggest correct settings for the continuous case.

Fix $\nu \geq 3$, and let the tree T_ν be a regular directed graph. The set V_ν of its vertices is in one-to-one correspondence with finite words in the alphabet $\mathcal{M} = \{1, 2, \dots, \nu\}$. The vertex $v_\emptyset$ is the origin of the tree. The kth generation is

$$G_k = \{v_I : I \in \mathcal{M}^k\},$$

so that

$$V_\nu = \bigcup_{k \geq 0} G_k.$$

The set of *children* of a vertex $v_I \in V_\nu$ is defined as $H_{v_I} = \{v_{I1}, \dots, v_{I\nu}\}$. We denote by $[v, w]$ the edge that links the vertices v and w. We define the set of edges E_ν of the tree T_ν in the following way: the edge $[v, w] \in E_\nu$ if and only if $w \in H_v$. Observe that if $[v, w] \in E_\nu$ then $[w, v] \notin E_\nu$ (T_ν is a directed graph).

Let $F : \bar{\mathbf{R}}_+^\nu \to \bar{\mathbf{R}}_+$ be a continuous function such that $F(0, 0, \dots, 0) = 0$ and $F(1, 1, \dots, 1) = 1$ (here $\bar{\mathbf{R}}_+ := [0, \infty)$ is the positive closed half-axis and $\bar{\mathbf{R}}_+^\nu := (\bar{\mathbf{R}}_+)^\nu$). We say that such a function F is *admissible.* In what follows, we consider only admissible functions. We understand $F(x_1, x_2, \dots, x_\nu)$ as a kind of nonlinear mean of the arguments $x_1, x_2, \dots, x_\nu$.

Let $n \geq 1$, and let ϕ be a function on $G_0 \cup \dots \cup G_n$. We say that ϕ is F-*harmonic* if

$$\phi(v_I) = F\big(\phi(v_{I1}), \phi(v_{I2}), \dots, \phi(v_{I\nu})\big)$$

for any $v_I \in G_0 \cup \dots \cup G_{n-1}$.

If A is a subset of vertices contained in G_n then we define the F-harmonic "measure" of A, denoted by $\omega_F(v, A)$, as the function defined in $G_0 \cup \dots \cup G_n$

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