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Adiabatic Decomposition of the ζ-Determinant and Scattering Theory

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1. Introduction and Statement of Results

Let $\mathcal{D}\colon C^\infty(M,S) \to C^\infty(M,S)$ be a compatible Dirac operator acting on sections of a Clifford bundle S over a closed manifold M of dimension n. The operator $\mathcal{D}$ is a self-adjoint operator with discrete spectrum $\{\lambda_k\}_{k\in\mathbb{Z}}$. The ζ-determinant of the Dirac Laplacian $\mathcal{D}^2$ is given by the formula

$$\det{}_\zeta \mathcal{D}^2 = e^{-\zeta'_{\mathcal{D}^2}(0)}, \tag{1.1}$$

where $\zeta_{\mathcal{D}^2}(s)$ is defined as follows:

$$\zeta_{\mathcal{D}^2}(s) = \sum_{\lambda_k\neq 0}(\lambda_k^2)^{-s} = \frac{1}{\Gamma(s)}\int_0^\infty t^{s-1}[\operatorname{Tr}(e^{-t\mathcal{D}^2}) - \dim\ker\mathcal{D}]\,dt. \tag{1.2}$$

This is a holomorphic function of s for $\Re(s) \gg 0$ and has the meromorphic extension to the complex plane with $s = 0$ as a regular point.

Let us consider a decomposition of M as $M_1 \cup M_2$, where M_1 and M_2 are compact manifolds with boundaries such that

$$M = M_1 \cup M_2, \qquad Y = M_1 \cap M_2 = \partial M_1 = \partial M_2. \tag{1.3}$$

In this paper we study the adiabatic decomposition of the ζ-determinant of $\mathcal{D}^2$, which describes the contributions in $\det_\zeta \mathcal{D}^2$ coming from the submanifolds M_1 and M_2. Throughout the paper, we assume that the manifold M and the operator $\mathcal{D}$ have product structures in a neighborhood of the cutting hypersurface Y. Hence, there is a bicollar neighborhood $N \cong [-1,1]_u \times Y$ of $Y \cong \{0\}\times Y$ in M such that the Riemannian structure on M and the Hermitian structure on S are products of the corresponding structures over $[-1,1]_u$ and Y when restricted to N, so that $\mathcal{D}$ has the following form:

$$\mathcal{D} = G(\partial_u + B) \ \text{ over } N. \tag{1.4}$$

Here u denotes the normal variable, $G\colon S|_Y \to S|_Y$ is a bundle automorphism, and B is a corresponding Dirac operator on Y. Moreover, G and B do not depend on u, and they satisfy

$$G^* = -G, \quad G^2 = -\mathrm{Id}, \quad B = B^*, \quad GB = -BG. \tag{1.5}$$

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