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Parameterizing Conjugacy Classes of Maximal Unramified Tori via Bruhat–Tits Theory

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0. Introduction

The main result of this paper is a uniform parameterization of the set of conjugacy classes of maximal unramified tori in a reductive p-adic group. This classification matches conjugacy classes of maximal unramified tori with certain equivalence classes that arise naturally from Bruhat–Tits theory. The motivation for this result comes from harmonic analysis; specifically, from J.-L. Waldspurger's papers [16; 17]. Using the parameterization scheme discussed in this paper, David Kazhdan and I [6] have been able to generalize some of the results of [17] in a uniform manner.

The Main Result. Let k denote a field with nontrivial discrete valuation ν. We assume that k is complete with perfect residue field $\mathfrak{f}$. Let $\bar{k}$ denote a fixed algebraic closure of k and let K denote the maximal unramified extension of k in $\bar{k}$. Let G denote the group of k-rational points of a reductive linear algebraic k-group $\mathbf{G}$ and let G° denote the group of k-rational points of the identity component $\mathbf{G}^{\circ}$ of $\mathbf{G}$. Let $\mathcal{B}(G)$ denote the (enlarged) Bruhat–Tits building of G°.

A subgroup of G is called an *unramified torus* when it is the group of k-rational points of a k-torus in $\mathbf{G}^{\circ}$ that splits over an unramified extension of k. In this paper we classify G-conjugacy classes of maximal unramified tori in G in terms of equivalence classes of pairs $(\mathsf{G}_F, \mathsf{T})$. Here F is a facet in the building, G_F is the connected reductive $\mathfrak{f}$-group associated to F, and T is an $\mathfrak{f}$-minisotropic maximal torus in G_F. (The torus T is called $\mathfrak{f}$-*minisotropic* when the maximal $\mathfrak{f}$-split torus in T coincides with the maximal $\mathfrak{f}$-split torus in the center of G_F.)

In more detail: Let I^t denote the set of pairs (F, T) where F is a facet in $\mathcal{B}(G)$ and T is a maximal $\mathfrak{f}$-torus in G_F. In Section 3.2 we define on I^t an equivalence relation, denoted $\sim$. In Section 3.3 we associate to each element $(F, \mathsf{T}) \in I^t$ a G-conjugacy class $\mathcal{C}(F, \mathsf{T})$ of maximal unramified tori in G. The set I^t is too

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