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# Linear Symmetric Determinantal Hypersurfaces

JENS PIONTKOWSKI

The question of which equations of hypersurfaces in the complex projective space can be expressed as the determinant of a matrix whose entries are linear forms is classical. In 1844 Hesse [He] proved that a smooth plane cubic has three essentially different linear symmetric representations. Dixon [Di] showed in 1904 that, for smooth plane curves, linear symmetric determinantal representations correspond to ineffective theta-characteristics—that is, ineffective divisor classes whose double is the canonical divisor. Barth [B] proved the corresponding statement for singular plane curves. The general case for any hypersurface was treated by Catanese [C], Meyer-Brandis [M-B], and Beauville [Be].

Any plane curve has a linear symmetric determinantal representation [Be, 4.4], but every linear symmetric determinantal surface is singular. By 1865 Salmon knew that such a surface of degree $n$ possesses in general $\binom{n+1}{3}$ nodes [S, p. 495], and Cayley [Ca] examined the position of these. Catanese [C] studied these surfaces with only nodes in a more general context. Here we are dealing mainly with the question of which combinations of singularities can occur on a linear symmetric determinantal cubic or quartic surface. For the cubics we find all their linear symmetric representations and obtain in particular the following theorem.

THEOREM. *There are four types of linear symmetric determinantal cubic surfaces with isolated singularities. The combinations of their singularities are given by the subgraphs of* $\tilde{E}_6$ *that are obtained by removing some of the white dots in Figure 1. In addition, all nonnormal cubics* (*with the exception of the union of a smooth quadric with a transversal plane*) *are linear symmetric determinantal cubics.*

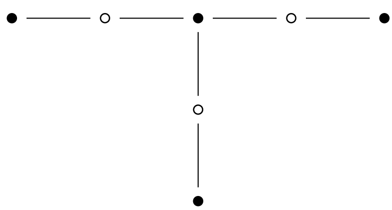

**Figure 1**

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