ON MS-FIBERINGS OF MANIFOLDS WITH FINITE SINGULAR SETS

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1. INTRODUCTION

Throughout the paper, we follow the notation and terminology of [1]. All manifolds are assumed to be closed, connected, and orientable, unless specifically stated otherwise, and they are taken in any of the categories Top, Diff, and PL. Fiberings shall be locally trivial and orientable. By a Hopf-type fibering of spheres, we mean a fibering $h: S^m \to S^q$ with fibre S^{m-q}, where $(m, q) = (3, 2), (7, 4)$, or $(15, 8)$. An MS-fibering $f: M^n \to N^p$ $(n > p)$ of manifolds is an open continuous map with the property that there exist closed, nonseparating sets A and B in M and N, respectively, satisfying the following conditions.

(i) $f(A) = B$ and $f \mid A: A \to B$ is a homeomorphism.

(ii) $f(M - A) \subset N - B$, and $f \mid (M - A): M - A \to N - B$ is a locally trivial fibration whose fibre is a manifold. The set A is referred to as the singular set of the MS-fibering f.

In [1], it is conjectured that if an MS-fibering $f: M^n \to N^p$ $(n > p)$ of manifolds has finite singular set A, then A consists of exactly two points provided

(1) f admits a spine fibering,

(2) N^p is the standard sphere S^p, and

(3) M^n is simply connected.

In this paper we prove that this conjecture is true in the following stronger form.

THEOREM A. *If* $f: M^n \to N^p$ $(n > p)$ *is an* MS-*fibering of manifolds with finite, nonempty singular set* A *and if* f *admits a spine, then* $\#(A) = 2$. *Moreover,* f *is* Top- *or* PL-*equivalent to the suspension of a Hopf-type fibering of spheres according as* f *is in* Top *or* PL (*modulo the Poincaré conjecture in dimensions* 3, 4).

Also, by means of the results in [1] one can easily prove the following assertion.

THEOREM B. *Let* $f: M^n \to N^p$ $(n > p)$ *be an* MS-*fibering of manifolds with singular set* A. *If* M^n *is* $([n/2] - 1)$-*connected and* $\#(A) = 2$, *then* f *admits a spine* (the square bracket denotes the greatest-integer function).

Combining these two theorems, we see that an MS-fibering $f: M^n \to N^p$ $(n > p)$ with singular set A is Top-equivalent to the suspension of a Hopf-type fibering if and only if M^n is $([n/2] - 1)$-connected and $\#(A) = 2$. One of the authors has discovered a large class of MS-fiberings $f: M^n \to S^p$ in Diff, with A-finite, nonempty and M^n $([n/2] - 1)$-connected [2]. It follows from our results that the suspension of a Hopf-type fibering is the only one among them admitting a spine.

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