HOMOTOPY EQUIVALENCE AND DIFFERENTIABLE PSEUDO-FREE CIRCLE ACTIONS ON HOMOTOPY SPHERES

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1. INTRODUCTION

This paper is concerned with differentiable pseudo-free circle actions on homotopy spheres, and the main result shows that each such action on a homotopy $(2n+1)$-sphere $(n \geq 1)$ may be mapped equivariantly, by a map of degree 1, onto a linear one on the $(2n+1)$-sphere with exactly one exceptional orbit. For the case $n = 1$, this is an easy consequence of a theorem of R. Jacoby [2], and for the case $n = 3$, it is contained in an earlier paper of Montgomery and Yang, though by a different proof [3]. The result will be used in a forthcoming paper to classify pseudo-free circle actions on spheres.

Except where it is contrarily stated, our study below is assumed to be in the differentiable category.

Let Σ^{2n+1} $(n \geq 1)$ be a homotopy $(2n+1)$-sphere on which there is a differentiable effective action of the circle group G such that all orbits are 1-dimensional. As usual, an orbit Gb in Σ^{2n+1} is called *exceptional* if the isotropy group G_b at b is not trivial. If there is at least one exceptional orbit and each exceptional orbit is isolated, the action is called *pseudo-free*. Suppose that a differentiable pseudo-free action of the circle group G on a homotopy $(2n+1)$-sphere $(n \geq 1)$ is given, and let $Gb_1, \cdots, Gb_k$ be the exceptional orbits in Σ^{2n+1}. Then for each $i = 1, \cdots, k$, the isotropy group G_{b_i} at b_i is a finite cyclic group $\mathbb{Z}q_i$ of order q_i for some integer $q_i > 1$, and the integers $q_1, \cdots, q_k$ are relatively prime to one another. In the following, we let

$$q = q_1 \cdots q_k,$$

which is an integer greater than 1.

Let G consist of complex numbers of absolute value 1, and let S^{2n+1} be the unit sphere in the unitary $(n+1)$-space $\mathbb{C}^{n+1}$. Then there exists a linear pseudo-free action of G on S^{2n+1}, given by the equation

$$g(z_0, z_1, \cdots, z_n) = (g^q z_0, gz_1, \cdots, gz_n).$$

Since $q > 1$, there exists exactly one exceptional orbit in S^{2n+1}, namely $|z_0| = 1$. The main theorem of this paper asserts the existence of an equivariant map of Σ^{2n+1} into S^{2n+1} of degree ± 1. (For the determination of the sign, see Theorem 2.) Notice that such a map induces a homotopy equivalence of the orbit space Σ^{2n+1}/G into the orbit space S^{2n+1}/G.

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