PIECEWISE-LINEAR CLASSIFICATION OF SOME FREE $\mathbb{Z}_p$-ACTIONS ON S^{4k+3}

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Let S^{2n+1} denote the unit $(2n + 1)$-sphere. Represent each of its points by a sequence $(c_0, \cdots, c_n)$ of complex numbers with $\sum |c_i|^2 = 1$. Let (S^1, S^{2n+2}) denote the S^1-action on S^{2n+1} given by the formula

$$c \cdot (c_0, \cdots, c_n) = (cc_0, \cdots, cc_n).$$

Let p be an odd prime, and let $\mathbb{Z}_p$ be the subgroup of S^1 generated by $\exp(2\pi i/p)$. Then (S^1, S^{2n+1}) induces a $\mathbb{Z}_p$-action on S^{2n+1}. Its orbit space

$$L^n(p) = S^{2n+1}/\mathbb{Z}_p$$

is the $(2n + 1)$-dimensional lens space. The purpose of this note is to study the piecewise-linear classification of all free $\mathbb{Z}_p$-actions on S^{4k+3} $(4k + 3 \geq 7)$ for which the orbit space is of the same simple homotopy type as $L^{2k+1}(p)$. Our main results follow.

THEOREM I. *Let* $\mathcal{S}ht(L^{2k+1}(p))$ *denote the set of equivalence classes of simple homotopy triangulations of* $L^{2k+1}(p)$. *If* $4k + 3 \geq 7$, *there exists an exact sequence of pointed sets*

$$0 \to L_{4k+4}(\mathbb{Z}_p)^\sim \to \mathcal{S}ht(L^{2k+1}(p)) \to [L^{2k+2}(p); G/PL] \to 0,$$

where $[L^{2k+1}(p); G/PL]$ *is the subgroup of* G/PL-*bundles on* $L^{2k+1}(p)$ *and* $L_{4k+4}(\mathbb{Z}_p)^\sim$ *is the reduced surgery obstruction group of C. T. C. Wall.*

THEOREM II. *There exists a one-to-one correspondence between the set*

$$\mathcal{S}ht(L^{2k+1}(p)) \times \left\{0, 1, \cdots, \frac{p-1}{2}\right\}$$

and the set of equivalence classes of free piecewise-linear $\mathbb{Z}_p$-*actions on* S^{4k+3} *whose orbit space has the same simple homotopy type as* $L^{2k+1}(p)$.

In the first section, we recapitulate some generalities about nonsimply-connected surgery, part of which has become folklore. We then carry out, in the second section, an elementary computation of the group $[L^{2k+1}(p); G/PL]$. Results of Sullivan are used in the proof of (2.1). Section 3 completes the proof of Theorem I. Section 4 is mainly a study of the homotopy classes of piecewise-linear homeomorphisms of a homotopy lens space. In the last section, we complete the proof of Theorem II.

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