CONCERNING THE ORDER STRUCTURE OF KÖTHE SEQUENCE SPACES, II

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1. INTRODUCTION

This paper investigates the order structure of the space $L(\lambda, \mu)$ of weakly continuous linear mappings of a sequence space λ into another sequence space μ. The weak topologies referred to here are those formed with respect to the respective α-duals of λ and μ, while the order structure of $L(\lambda, \mu)$ is that generated by the positive mappings in $L(\lambda, \mu)$ when λ and μ are equipped with their natural order. We shall use several important results concerning the algebraic structure of $L(\lambda, \mu)$ that are found in the fundamental paper [6] of G. Köthe and O. Toeplitz and in the work of H. S. Allen (see [1] and Chapter 6 of [4]). We shall also use the results and terminology of our earlier work [8].

2. PRELIMINARY MATERIAL

Throughout this paper we shall assume that λ and μ are real sequence spaces containing the space ϕ of sequences with only a finite number of nonzero components. The positive cones of sequences with nonnegative components in λ and μ will be denoted systematically by K_λ and K_μ, respectively; K'_λ and K'_μ will denote the corresponding dual cones in the α-duals λ^* and μ^* of λ and μ, respectively. We shall always assume that λ is a solid; that is, if $|x| \leq |y|$ and $y \in \lambda$, then $x \in \lambda$ (here $|x| = (|x_i|)$ denotes the lattice-theoretic absolute value of x in λ). We refer the reader to [5] and [8] for further details concerning the topological and order-theoretic properties of sequence spaces.

A *matrix transformation* on λ into μ is an infinite matrix $A = (a_{ij})$ with the following properties:

(M_1) For each $x = (x_j) \in \lambda$, the series $\Sigma_{j=1}^{\infty} a_{ij}x_j$ converges absolutely for each i.

(M_2) For each $x = (x_j) \in \lambda$, the equation $y_i = \Sigma_{j=1}^{\infty} a_{ij}x_j$ defines an element $y = (y_i)$ of μ.

If $A = (a_{ij})$ is a matrix transformation on λ into μ, then the mapping $y = Ax$ defined by (M_2) is clearly a linear mapping of λ into μ. On the other hand, if T is a linear mapping of λ into μ and if there exists a matrix transformation A of λ into μ such that $Tx = Ax$ for all $x \in \lambda$, then T is *represented* by A. If T is represented by a matrix transformation A, then A is unique, since λ contains the "unit vectors" $e^{(k)} = (\delta_{ik}: i = 1, 2, \cdots)$ (δ_{ik} denotes the Kronecker delta). The following result, essentially due to G. Köthe, O. Toeplitz, and H. S. Allen, is stated here in a form convenient for our purposes.

(2.1) PROPOSITION. *The following conditions on a linear mapping* T *of* λ *into* μ *are equivalent:*

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