Notre Dame Journal of Formal Logic
Volume VII, Number 3, July 1966

A SIMPLE VERSION OF THE GENERALIZED CONTINUUM HYPOTHESIS

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An understanding of the use of the following expressions of Zermelo-Fraenkel-Skolem set theory without the axiom of choice is presupposed: '$\mathsf{S}x$' ('The set of all subsets of x'), '2' ('The set whose only members are the empty set and the set whose only member is the empty set'), 'x^y' ('The set of all functions from y into x'), '$x \approx y$' ('There is a one-to-one correspondence between x and y'), and 'x is finite'.

Definition 1. $x \preccurlyeq y$ *if and only if there is a* z *in* $\mathsf{S}y$ *such that* $x \approx z$;
$x \prec y$ *if and only if* $x \preccurlyeq y$ *and not* $y \preccurlyeq x$.

Definition 2. **AC** *if and only if, for any* x, *if the members of* x *are non-empty and disjoint, then there is an* s *such that, for any* m *in* x, *there is exactly one* d *in both* m *and* s;
GCH *if and only if, for any* x, *if* x *is not finite, then there is no* y *such that both* $x \prec y$ *and* $y \prec 2^x$.

The following theorems are well-known:

Theorem 1. **AC** *if and only if, for any* x *and* y, *either* $x \preccurlyeq y$ *or* $y \preccurlyeq x$.

Theorem 2. *If either* x *or* y *is finite, then either* $x \preccurlyeq y$ *or* $y \preccurlyeq x$.

Theorem 3. *If* **GCH**, *then* **AC**.

From theorems 1 and 2 we have

Theorem 4. **AC** *if and only if, for any* x *and* y, *if both* x *and* y *are not finite, then either* $x \preccurlyeq y$ *or* $y \preccurlyeq x$.

This version of the axiom of choice suggests a nearly identical version of the generalized continuum hypothesis.

Definition 3. **G** *if and only if, for any* x *and* y, *if both* x *and* y *are not finite, then either* $x \preccurlyeq y$ *or* $\mathsf{S}y \preccurlyeq x$.

Theorem 5. **G** *if and only if* **GCH**.

Received August 15, 1963