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ON MODAL RENDERINGS OF INTUITIONISTIC PROPOSITIONAL LOGIC

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The intuitionistic propositional calculus (IPC) of Heyting[1] rests upon the following eleven axioms:

(A1) $p \rightarrow (p \wedge p)$
(A2) $(p \wedge q) \rightarrow (q \wedge p)$
(A3) $(p \rightarrow q) \rightarrow [(p \wedge r) \rightarrow (q \wedge r)]$
(A4) $[(p \rightarrow q) \wedge (p \rightarrow r)] \rightarrow [p \rightarrow r]$
(A5) $q \rightarrow (p \rightarrow q)$
(A6) $[p \wedge (p \rightarrow q)] \rightarrow q$
(A7) $p \rightarrow (p \veebar q)$
(A8) $(p \veebar q) \rightarrow (q \veebar p)$
(A9) $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \veebar q) \rightarrow r]$
(A10) $\neg p \rightarrow (p \rightarrow q)$
(A11) $[(p \rightarrow q) \wedge (p \rightarrow \neg q)] \rightarrow \neg p$

Here the symbols '$\rightarrow$', '$\wedge$', '$\veebar$', and '$\neg$' are used for intuitionistic implication, conjunction, disjunction, and negation, respectively.

Moreover, there are certain theses which Heyting in his book of 1956 specifically and explicitly rejects as intuitionistically unacceptable:

(U1) $(p \vee q) \rightarrow (p \veebar q)$ [See p. 97.]
(U2) $\sim p \rightarrow \neg p$ [See pp. 18-19, 97-98.]
(U3) $p \veebar \neg p$ [See p. 99.]
(U4) $\neg \neg p \rightarrow p$ [See p. 99.]
(U5) $(p \rightarrow q) \veebar (q \rightarrow p)$ [See p. 99.]
(U6) $\neg (p \wedge q) \rightarrow (\neg p \veebar \neg q)$ [See p. 100.]
(U7) $(\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$ [See p. 101.]
(U8) $\neg \neg (p \veebar q) \rightarrow (\neg \neg p \veebar \neg \neg q)$ [See p. 101.]

The symbols '$\sim$', '&', and 'v' will be used for "ordinary" (non-intuitionistic) negation, conjunction, and disjunction, respectively; and '$\supset$' will be used below for material implication.

Various "dictionaries" for "translating" statement schemata of IPC into the vocabulary of Lewis' systems of strict implication have been or can

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