

MODAL LOGIC WITH FUNCTORIAL VARIABLES AND A CONTINGENT CONSTANT

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The World as a Propositional Constant

1. The present section is by Prior; the two which follow it, by Meredith, Meredith's sections were originally produced in 1953 and circulated among colleagues; subsequent references to them in the literature, e.g. in [3], [4], [5], [6] and [7], may be clarified if these two notes are now made more widely available. They were provoked by Łukasiewicz's development of the modal system which he presented in [2]. The importance of this system as a limiting case has been made clear by Smiley in [9]; a number of commentators have noted its intuitive peculiarities. Meredith was interested in it as a first attempt to incorporate functorial variables in a modal system, and sought in the system $(C, \Gamma, 0, \delta, p)$ below to incorporate the same feature in a more normal type of modal logic, namely Lewis's S5. This system is equivalent to S5 supplemented by the qualified law of extensionality $C\Gamma EpqC\delta p\delta q$ (Meredith takes over Łukasiewicz's symbol Γ for necessity and Δ for possibility).

2. The system $(C, \Gamma, 0, n, \delta, p)$ introduces the more original feature of a constant n to represent "the world" in the Wittgensteinian sense of "everything that is the case." Its most distinctive feature is the law $Cp\Gamma Cnp$, "What is true is necessarily implied by the totality of what is the case"—necessarily because this totality is equivalent to a conjunction of which all true propositions are conjuncts, and we have $\Gamma CKpqp$. In a sense, of course, unless all truths are necessary, the totality of what is the case might not have contained (and so implied) the given truth p ; but in the symbol n , "the totality of what is the case" is not given by this description of it but given simply as the actual totality of what *is* the case.

Meredith's proof, in his second item, of the independence of this fundamental law $Cp\Gamma Cnp$, is instructive. To distinguish a contingent truth from a necessary one we need two possible worlds, a contingent truth holding in one of them only and a necessary truth in both. Two such worlds generate four truth-values, "truth in both", "true in the actual world but not in the other", "true in the other world but not in the actual one", "true in

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