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STUDIES ON THE AXIOM OF COMPREHENSION

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In a previous paper in this journal (see [1]) I constructed a model **M** of a set theory such that the axiom

$$(Ey)\ (x)\ (x \epsilon y \leftrightarrow \phi(x))$$

is valid, where $\phi(x)$ may contain some parameters $z_1, \ldots, z_n$ and is built by conjunction and disjunction alone from atomic propositions $u \epsilon v$, where u and v are any two of $x, z_1, \ldots, z_n$. In particular it was also allowed that $\phi(x)$ is just a propositional constant *0* (false) or *1* (true). In this note I shall add a few further results concerning models of set theories for which certain axioms are given. In § 1 I first mention some general forms of the comprehension axiom and then prove a further theorem on the model in [1]. In § 2 I give a new proof of a result in [2], where a certain 3-valued logic was considered. In § 3 I show some further examples of models of set theories in ordinary 2-valued logic.

§1.

We may consider 3 forms of the axiom of comprehension. The first is that partially treated in [1], although I prefer to write it here in the form

$$(1)\quad (z_1) \ldots (z_n)\ (Ey)\ (x)\ (x \epsilon y \leftrightarrow \phi(x, z_1, \ldots, z_n)),$$

where ϕ is either a propositional constant or built from atomic expressions $u \epsilon v$ by negation, conjunction and disjunction and there are no further variables in ϕ than $x, z_1, \ldots, z_n$. The second form is

$$(2)\quad (z_1) \ldots (z_m)(Ey)(x)(x \epsilon y \leftrightarrow \prod_{u_1} \ldots \prod_{u_n} \phi(x, z_1, \ldots, z_m, u_1, \ldots, u_n)),$$

where ϕ as before is built by the connectives of the propositional calculus while each $\prod_{u_r}$ means either universal or existential quantification with regard to u_r. It may be advantagous also to consider a third form

$$(3)\quad (Ey)(x)(x \epsilon y \leftrightarrow \prod_{u_1} \ldots \prod_{u_n} \phi(x, u_1, \ldots, u_n)),$$

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