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S1° AND BROUWERIAN AXIOMS

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Using the notation of [1] we discuss the effect of adding one or more Brouwerian axioms B_n : $\mathfrak{C}pL^nMp$ $(n \geq 1)$ to S1°. We call a set of one or more such axioms *sufficient* or *insufficient* according as its addition to S1° does or does not yield S5. The means of proof available in S1° will be everywhere pre-supposed.

Theorem I. No set of axioms of the form B_{2k+1} $(k \geq 0)$ *is sufficient.*

Proof is by the matrix:

K	1 2 3 4	N	M
* *1*	1 2 3 4	*4*	*1*
2	2 2 4 4	*3*	*3*
3	3 4 3 4	*2*	*2*
4	4 4 4 4	*1*	*4*

which satisfies S1° and all B_{2k+1} but rejects $\mathfrak{C}pMp$.

Theorem II. Any pair B_1, B_{2k} $(k \geq 1)$ *is sufficient.*

From B_1 we have (1) $\mathfrak{C}L^2pp$; from B_{2k} we have (2) $\mathfrak{C}LpL^{2(k+1)}p$; $k + 1$ applications of syllogism to (1), (2) give $\mathfrak{C}Lpp$ and so S1. But, as is known, $\{S1, B_{2k}\} = S5$.

Theorem III. For all m, n greater than 2, if m and n are co-prime, then B_{m-1}, B_{n-1} *are sufficient.*

If m and n are co-prime, there are positive integral r and s such that $rm = sn \pm 1$ and so $rm - 1 = sn - 1 \pm 1$. But B_{m-1} yields $\mathfrak{C}LpL^{m+1}p$ if $m > 2$, and so, by replacement, B_{rm-1} for all r not less than unity. Similarly from B_{n-1} we can obtain all B_{sn-1}, and proceed as follows:

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