Notre Dame Journal of Formal Logic
Volume XIII, Number 2, April 1972
NDJFAM

AN EQUATIONAL AXIOMATIZATION OF ASSOCIATIVE NEWMAN ALGEBRAS

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An associative Newman algebra is a Newman algebra[1] in which the binary multiplicative operation $\times$ is associative for all elements belonging to the carrier set of the considered system. In [2], p. 265 and p. 271, Theorem 5 and Example E10, Newman has established that such an algebraic system is a proper extension of his complemented mixed algebra,[2] and that it is a direct join of an associative Boolean ring with unity element and a Boolean lattice (i.e. a Boolean algebra). Moreover, he has shown there that this system can be constructed by an addition of a rather weak formula, viz. *K1* given in section 1 below, as a new postulate, to the axiom-system formulated in [2] of Newman algebra. On the other hand, it is almost self-evident that an associative Newman algebra is not necessarily a Boolean algebra.

In this note it will be shown that the addition of formula *K1* mentioned above, as a new postulate, to the set of axioms of system $\mathfrak{B}$ discussed in [3] allows us to construct a very simple and compact equational axiom-system for associative Newman algebra.

1 We define a system under consideration as follows:

Any algebraic system

$$\mathfrak{D} = \langle B, =, +, \times, - \rangle$$

with *one binary relation* $=$, *two binary operations* $+$ *and* $\times$, *and one unary operation* $-$, *is an associative Newman algebra, if it satisfies the postulates*

1. An acquaintance with the the papers [2] and [3] is presupposed. An enumeration of the formulas used in this note is a continuation of the enumeration which is given in [3]. As in that paper, the properties of "even" and "odd" elements will be not discussed in this note, and the axioms *A1*-*A11* given below will be used mostly tacitly in the deductions.

2. I.e., of Newman algebra, *cf.* [3].

Received April 18, 1971