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A NEW FORMALIZATION OF NEWMAN ALGEBRA

BOLESŁAW SOBOCIŃSKI

In [6][1] M. H. A. Newman constructed and investigated an algebraic system whose two basic binary operations are + and $\times$,[2] and which, as he has proved, is a direct join of a non-associative Boolean ring with unity element and a Boolean lattice, *i.e.* a Boolean algebra. In [7], p. 28, Newman calls this system a *complemented mixed algebra,* but in Birkhoff's [2] and [3], p. 48, it is called Newman algebra. The latter name will be used throughout this paper. Besides the property which is mentioned above, in [6] it has been proved that for all elements of the carrier set of any Newman algebra the additive operation + is commutative and associative, but not necessarily idempotent or nilpotent, and that the multiplicative operation $\times$ is idempotent and commutative, but not necessarily associative.

The main aim of this paper is to show that Newman algebra can be formalized as an equational system. For this end in section **1** below two definitions, (**A**) and (**B**), of two systems, $\mathfrak{A}$ and $\mathfrak{B}$ respectively, of the Newman algebras are given, and in section **2** it will be proved that these systems are inferentially equivalent, if their respective carrier sets A and B are the same, *i.e.* $A = B$, or these systems are inferentially equivalent up to isomorphism, if their carrier sets have only the same cardinality, *i.e.* $\mathrm{card}(A) = \mathrm{card}(B)$. Since definition (**A**) of $\mathfrak{A}$ is an obviously equivalent modification of a formalization of Newman algebra given in [1], p. 4, [2], p. 155, and [3], p. 49, and since (**B**) defines $\mathfrak{B}$ as an equational system, our claim will be justified. In section **3** it will be proved that in the field of $\mathfrak{B}$ the set of its proper algebraic postulates is inferentially equivalent to another set containing a very small number of axioms. Finally, in section **4** the mutual independence of the axioms belonging to the sets mentioned above will be established.

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1. An acquaintance with the papers [6], [7] and one of [1], [2] or [3] is presupposed. *Cf.* also [8] and [4].

2. In the papers mentioned in note 1 "*ab*" is used instead of "$a \times b$."

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