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FOUR-VALUED TABLES AND MODAL LOGIC

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1. It would be extremely convenient to be able to use the four-valued tables devised by W. T. Parry[1] as a decision procedure in systems of propositional modal logic such as Lewis' S1-S5, in much the same way that truth tables are used in assertoric propositional logic. Furthermore, if one attempts to use the four-valued tables in this way it will be found that all theorems of such systems receive designated values for every value of the variables when one uses the appropriate table for the system; e.g. all S3 theorems receive designated values using the S3 tables. Furthermore, no known formula which is not intuitively valid receives designated values for every value of its variables, if one makes appropriate allowances for the difference in the systems S1-S5, and the four-valued aspect of the system, discussed below.[2]

As against these facts, which mean that in practice one runs into no difficulties in using the four-valued tables as quasi-truth tables, there is a result due to James Dugundji[3] which shows that there can be no matrix of a finite number of elements which satisfies those, and only those formulas which are provable in the systems S1-S5.

In what follows I wish to discuss Dugundji's paper, showing why the practical success of the tables is not surprising despite Dugundji's result, and how the tables can provide a decision method which is in practice reliable, as well as being infinitely more convenient than other proposed procedures.

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1. *Cf.* Lewis and Langford, *Symbolic Logic*, Second Edition, New York, Dover Publications (1957), Appendix II.
2. This statement is based on considerable experience: using techniques partly described in my paper "Doing Logic by Computer," *Notre Dame Journal of Formal Logic*, vol. 10 (1969), pp, 150-162, I have tested a large number of modal statements by a computer program which uses the four-valued tables as a decision procedure.
3. "Note on a Property of Matrices for Lewis and Langford's Calculi of Propositions," *The Journal of Symbolic Logic*, vol. 5 (1940), pp. 150-151.

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