Notre Dame Journal of Formal Logic
Volume XI, Number 3, July 1970

LOCATING VERTICES OF TREES

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Let X be a nonempty set and let R and S be binary relations on X. Let x, y, x_1, x_2, y_1, y_2 be arbitrary elements of X, then, where $\mathrm{R}(R)$ is the range of R and ω is the set of nonnegative integers, $\langle X, R, S\rangle$ is called a *(dyadic ordered) tree* if the following hold:

(1) If x_1Ry and x_2Ry, then $x_1 = x_2$.
(2) For each $x \in X$, xRy for at most two y.
(3) X - $\mathrm{R}(R)$ is a unit set, $\{x_0\}$.
(4) y_1Sy_2 iff (a) $y_1 \neq y_2$, (b) $y_2 \not{S} y_1$, and (c) for some $x \in X$, both xRy_1 and xRy_2.
(5) There exists a function l: $X \to \omega$ with the properties: (a) $l(x_0) = 0$ and (b) if xRy, then $l(y) = l(x) + 1$.

This definition, with minor modifications, is essentially the one given in [1].

If $\langle X, R, S\rangle$ is a tree, then the elements of X are called *points* or *vertices*. If xRy holds for a unique $y \in X$, x is called a *simple point*; if xRy holds for two distinct y, x is called a *junction point*. Whenever xRy then y is said to be an *immediate successor of* x. The relation S, in effect, selects one of the two immediate successors of a junction point. Thus if xRy_1, xRy_2 and y_1Sy_2, we say that y_1 is the *left successor* and y_2 the *right successor of* x.

$l(x)$ is called the *level of* x. l_n will denote the set of vertices of level n, $n \in \omega$. Each l_n has at most 2^n vertices; hence for any tree $\langle X, R, S\rangle$, X must be countable. Note that $\langle X, R, S\rangle$ has no junction points iff l is one-one iff $S = \emptyset$.

A *path* of a tree $\langle X, R, S\rangle$ is a finite sequence $[a_0, a_1, \ldots, a_n]$ or a denumerable sequence $[a_0, a_1, \ldots, a_n, \ldots]$ with the properties:

(1) for each a_k appearing in the sequence, $a_k \in X$ and
(2) if a_{k+1} also appears in the sequence, then a_{k+1} is an immediate successor of a_k.

Received May 7, 1969