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PROPOSITIONAL SEQUENCE-CALCULI FOR INCONSISTENT SYSTEMS

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Contradictions arise mostly at the beginning and at the end of a theoretical construction. If the meaning of words is not fixed a proposition and its negation can both be true. And, as it is well known, a theory in spite of the axiomatic fixation of its concepts must contain inconsistencies if its means of expression are sufficiently rich. If classical—but also intuitionistic—propositional logic is used as the basic logical frame of a theory the deduction of a contradiction produces its complete trivialization: every proposition is deducible in it, *ex falso sequitur quodlibet.* The minimal logic avoids this logical principle but from a contradiction we can deduce in it the negation of every proposition. By weakening the classical logic S. Jaśkowski (*cf.* [6]) and specially Newton C. A. da Costa have built propositional calculi which enable us to overcome this difficulty. By the way these systems contain logical laws which Hegelians in spite of their famous rejection of the principle of contradiction must acknowledge as valid.

Da Costa has built a hierarchy of propositional calculi $\mathbf{C}_n(1 \leq n \leq \omega)$ whose decidability has not been settled yet. We tried first to solve this problem constructing equivalent sequence-calculi and proving the corresponding cut-theorems. But this new hierarchy which we called $\mathbf{CG}_n(1 \leq n \leq \omega)$ showed some restrictions which are only justified from an intuitionistic point of view. By dropping these restrictions we have constructed a new hierarchy $\mathbf{WG}_n(1 \leq n \leq \omega)$ of decidable calculi with the same essential properties of the $\mathbf{C}_n$.*

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