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SOME PROOFS OF RELATIVE COMPLETENESS IN MODAL LOGIC

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In this paper we adopt for a number of modal systems a method of proving completeness used in [1] (to prove the completeness of S4 relative to T).[1] We prove the completeness of S7, S8 relative to S3, of S3, S6 relative to S2 and of S2 relative to E2.

The system E2 was proposed by E. J. Lemmon[2] and can be axiomatized as follows;

A1 *If* α *is a* **PC**-*tautology then* $\vdash \alpha$
A2 $Lp \supset p$
A3 $L(p \supset q) \supset (Lp \supset Lq)$

(with uniform substitution or replaced by the equivalent schemata)

A4 $\vdash \alpha \supset \beta \rightarrow \vdash L\alpha \supset L\beta$
A5 (**MP**) $\vdash \alpha$, $\vdash \alpha \supset \beta \rightarrow \vdash \beta$

E2 is in fact the system T (as axiomatized in [4], pp. 533-535) without the rule of necessitation. We obtain S2 by replacing A4 by,

A6 $\vdash L(\alpha \supset \beta) \rightarrow \vdash L(L\alpha \supset L\beta)$

and adding;

A7 *If* α *is a* **PC**-*tautology or an axiom then* $\vdash L\beta$[3]

For S3 we replace A6 and A3 by

A8 $L(p \supset q) \supset L(Lp \supset Lq)$

For S6 we add to S2, A9 MMp
For S7 we add to S3, A10 MMp
For S8 we add to S3, A11 $LMMp$
where A7 does not apply to A9 - A11.

An account of validity for these systems has recently been given by Saul Kripke [7] and it is essentially this account we shall use. We define an E2 model as an ordered quadruple $<VWRx_1>$ where W is a set of objects (worlds), $x_1 \in W$ and R a quasi-reflexive relation over W. By this is meant that for any $x_i \in W$ if any x_jRx_i then x_iRx_i. Members x_i of W such that x_iRx_i are called normal.[4] Quasi-reflexiveness ensures that no world is

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