

ARITHMETIC AS A STUDY OF FORMAL SYSTEMS

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The notion of formal system appearing in the works of Curry seems to have been somewhat misunderstood. The purpose of this paper* is to help clarify this notion. I shall do this by considering elementary, almost trivial examples of formal systems of arithmetic and by showing how these systems can be considered a natural result of the increasing need, as the theory of arithmetic progressed, to find procedures for answering new kinds of questions. I shall also discuss some details of some of these examples, for example the difference between two kinds of metatheoretic implication relative to a formal system.

1 *Informal arithmetic and the need for increasing rigor* The earliest numbers were marks of some sort: notches in a stick or strokes on an animal skin. The idea of numbers in the modern sense arose when it was observed that no matter how many marks one has, it is always possible to add another one, so that there are infinitely many (natural) numbers.

With the growth of the theory about these numbers, a need arose for a more complete description of the nature of these numbers. This was especially true when, in the last century, the theory of classical analysis was developed entirely from the properties of natural numbers. Hence, at the end of the century, several such descriptions appeared, of which the best known is the set of axioms used by Peano. These axioms, in which the primitive ideas are 0 (an obvious modification can be made to begin with 1 instead of 0), representing the empty sequence of marks, |, representing the operation of adding one more mark, and =, a relation between numbers meaning that the marks of the two numbers can be paired in a 1 - 1 manner, are the following:

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