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NOTE ON THE USE OF SEQUENCES IN 'LOGICS AND LANGUAGES'

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The purpose of this note is to remedy a defect in the set-theoretical basis of my book *Logics and Languages* [1]. In most accounts of formal languages, expressions are regarded as strings of symbols. Thus in the propositional calculus

(1) $((p \supset q) \supset r)$

is a string whose first and second members are (, whose third member is p and so on. Two things are not usually made clear. First, what a string is; and second, what (and p and so on are. In [1] I tried to tackle these problems by giving a purely set-theoretical analysis of the expressions of a formal language. It has, however, been drawn to my attention that there are certain infelicities, if not actual errors, in my definitions, and it is the purpose of this note to remedy the major one of these and, hopefully, arrive at an adequate statement of a set-theoretical representation of expressions.

Beginning with the standard notions of set theory we define the *ordered pair* (x, y) as $\{\{x\}, \{x, y\}\}$. We now define a *relation* as a class of ordered pairs and a *function*, f, as a relation such that, if $(x, y) \in f$ and $(x, z) \in f$, then $y = z$. We let $f(x)$ denote the unique y such that $(x, y) \in f$. The *domain* of f is the set of all x such that for some y, $(x, y) \in f$, and the *range* of f is the set of all y such that for some x, $(x, y) \in f$.

A *sequence* is a function whose domain is an initial segment of the natural numbers. We shall take the natural numbers to be the set $\{1, 2, 3, \ldots\}$ and not to include 0. Sequences can be finite or infinite. If i is a natural number and x is a sequence[1] then $x(i)$ is usually written x_i. If x is an infinite sequence then its domain is the whole of the natural numbers.

1. I am using the letters 'x', 'y', etc. to range quite generally over classes. Thus they range *inter alia* over functions. The letters 'f', 'g', etc. are usually reserved for functions only, but the important point is that functions are merely classes of a special kind. When x is a function, x_i denotes its value for the argument i.

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