Notre Dame Journal of Formal Logic
Volume XVI, Number 3, July 1975
NDJFAM

A NOTE ON $\mathcal{P}$-ADMISSIBLE SETS WITH URELEMENTS

JUDY GREEN

In [2] Barwise states that although the introduction of urelements into Zermelo-Fraenkel set theory is redundant, their introduction into the weaker Kripke-Platek theory for admissible sets is not. In this note* we will show that their introduction into the intermediate theory of power set admissible sets is once again redundant since all $\mathcal{P}$-admissible sets with urelements are of the same form as $\mathcal{P}$-admissible sets, i.e., $\mathsf{V}_M(\kappa) = \mathsf{H}_M(\kappa)$ where κ is a strong limit cardinal and $\kappa = \beth_\kappa$.

We assume familiarity with the formulation of the theory **KPU** (Kripke-Platek with urelements) and the language in which it is formulated (see [2]). We also assume familiarity with the hierarchy of set theoretic predicates due to Lévy [5], and the primitive recursive set functions of Jensen and Karp [4]. We expand the notation of [2] as follows:

Definition: A structure $\mathfrak{A}_{\mathfrak{M}} = (\mathfrak{M}; A, E, P, \ldots)$ for the language $\mathsf{L}(\epsilon, \mathcal{P}, \ldots)$ consists of

(1) a structure $\mathfrak{M} = \langle M, \ldots \rangle$ for the language L,
(2) a nonempty set A disjoint from M,
(3) a relation $E \subseteq (M \cup A) \times A$ to interpret ϵ,
(4) a function P from A into A to interpret $\mathcal{P}$, and
(5) other functions, relations, and constants on $M \cup A$ which interpret the other symbols in $\mathsf{L}(\epsilon, \mathcal{P}, \ldots)$.

In the language $\mathsf{L}(\epsilon, \mathcal{P}, \ldots)$ variables are distinguished to allow quantification over M (urelements), A (sets), and $A \cup M$. The variables used are, respectively: $p, q, r, \ldots$; $a, b, c, d, \ldots$; and $x, y, z, \ldots$.

Definition: The theory $\mathcal{P}$-**KPU** consists of the universal closures of the axioms of

extensionality: $\forall x(x \in a \leftrightarrow x \in b) \rightarrow a = b$,

*Research supported by the Rutgers University Research Council.

Received July 15, 1974