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A NOTE ON THE TRUTH-TABLE FOR $p \supset q$

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A didactic problem which inevitably confronts the teacher of elementary symbolic logic is the justification of the truth-table for $p \supset q$. Every teacher has encountered the inquisitive, contentious student who refuses to admit $\mathsf{F} \supset \mathsf{T}$ and $\mathsf{F} \supset \mathsf{F}$ are true. The justifications commonly found in textbooks, for one reason or another, all fall short of satisfying this student.

For instance, Jan Łukasiewicz, in order to justify the truth-table, made the following use of the obviously true proposition, If x is divisible by 9, then x is divisible by 3:

> This implication is true for all the values of the numerical variable x. Hence on substituting $x/16$ we should obtain a true sentence. The substitution yields:
>
> If 16 is divisible by 9, then 16 is divisible by 3.
>
> We have thus obtained an implication with a false antecedent and a false consequent. In view of such examples we agree $C00 = 1$, i.e., that *an implication with a false antecedent and a false consequent is true*. By substituting $x/15$ we obtain:
>
> If 15 is divisible by 9, then 15 is divisible by 3.
>
> Now the antecedent is false and the consequent true. We therefore agree that $C01 = 1$, i.e., that *an implication with a false antecedent and a true consequent is true*.
>
> By substituting $x/18$ we obtain an implication with a true antecedent and a true consequent:
>
> If 18 is divisible by 9, then 18 is divisible by 3.
>
> Consequently, we agree that $C11 = 1$, i.e., that *an implication with a true antecedent and a true consequent is true.*[1]

Unfortunately, the contentious student can employ an analogous argument to make a prima facie case for evaluating $\mathsf{F} \supset \mathsf{F}$ and $\mathsf{F} \supset \mathsf{T}$ (as well as $\mathsf{T} \supset \mathsf{F}$) as false. Consider the obviously false proposition: If Fig. *ABCD* is a square, it has only three sides. Now consider the following three figures:

1. Jan Łukasiewicz, *Elements of Mathematical Logic*, Pergamon Press, New York (1963), p. 26.

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