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COMBINATORY AND PROPOSITIONAL LOGIC

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The relationship between combinatory and propositional logic is dealt with at length in [1] and tangentially in [2]. The present paper adds nothing essentially new to previous results. It does, however, offer a straightforward procedure, which for any λ-expression in normal form will either lead to its propositional correspondent or determine that this is null. Section 1 presents the hypothesis upon which the correspondence between λ-expressions and propositional formulae is based; our translation procedure is described in section 2, and illustrated in section 3.

1 Hypothesis. In dealing with λ-expressions we assume Church's rules and conventions as given in [3]. With respect to propositional formulae, 'Λ' denotes the null class of formulae, and 'Γ' is used for C. A. Meredith's operator D: 'ΓPQ' denotes the most general result that can be obtained when *Modus Ponens* is applied with P, or some substitution in it, as major premiss, and Q, or some substitution in it, as minor premiss. '$\sim$' denotes correspondence between a λ-expression and a propositional formula. Our basic hypothesis is the following.

Hypothesis *Where L, M and N are λ-expressions, P, Q and R are propositional formulae, and Σ is an operation under the substitution rule which may be null.*

1. *Let $L \sim P$, then for M with no free variables in common with L, and all N, Q, R. If $M \sim Q$, $LM = N$, and $N \sim R$, then either $\Gamma PQ = \Sigma R$ or $\Gamma PQ = \Lambda$.*

2. *Let $N \sim \Lambda$, then for L, M with no free variables in common, and all P, Q. If $L \sim P$, $M \sim Q$, and $LM = N$, then $\Gamma PQ = \Lambda$.*

The need for the two cases under the first section

(a) $L \sim P,\ M \sim Q,\ LM = N$ and $\Gamma PQ = \Sigma R$ for Σ non-null
(b) $L \sim P,\ M \sim Q,\ LM = N,\ N \sim R$ and $\Gamma PQ = \Lambda$

is unfortunate but unavoidable. With respect to (a): if for $L \sim P$ we take

$$\lambda abcd\,.\,ac(bd) \sim CCpCqrCCsqCpCsr$$

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