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INFINITE SERIES OF T-REGRESSIVE ISOLS

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1 Introduction.* Let E denote the collection of all non-negative integers (numbers), Λ the collection of all isols, Λ_{R} the collection of all regressive isols, and Λ_{ZR} the collection of all cosimple regressive isols. Infinite series of regressive isols were defined by J. C. E. Dekker in [4]; A. Nerode in [14] associated with every recursive function $f(x)$ an extension of $f(x)$ to a mapping $D_f(X)$ on Λ. In [1], J. Barback showed that $D_f(X)$ for f an increasing recursive function and $X \epsilon \Lambda_R$ can be represented as an infinite series. Universal isols were introduced by E. Ellentuck in [6].

The collection Λ_{TR} of T-regressive isols was introduced in [8]. There a result was proved concerning an equality between infinite series of T-regressive isols; viewing the extension of a recursive combinatorial function to Λ_R in terms of infinite series, this result led to a proof that T-regressive isols are universal. In the present paper, three further results are obtained concerning equalities and inequalities between infinite series of isols when T-regressive isols are involved. As applications of Theorem 1 below, we obtain new proofs of several previously known results concerning extensions of recursive functions to Λ_R. Theorem 3 below is used by M. Hassett in obtaining his main result of [10].

2 Preliminaries. We recall from [4] the definition of an infinite series of isols, $\sum_{T} a_n$, where T denotes an infinite regressive isol and a_n denotes a function from E into E:

$$\sum_{T} a_n = \mathrm{Req} \sum_{0}^{\infty} j(t_n, \nu(a_n))$$

where $j(x, y)$ is a recursive function mapping E^2 one-to-one onto E, t_n is any regressive function ranging over a set in T, and for any number n, $\nu(n) = \{x \mid x < n\}$. By results in [4], $\sum_{T} a_n$ is an isol and is independent of the choice of the regressive function whose range is in T. In [2], J. Barback studied infinite series of the form $\sum_{T} a_n$ where $T \leq^* a_{n-1}$. The relation

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