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MODALITY AND PREFERENCE RELATION

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Halldén's system $\mathcal{A}$ on logic of preference contains the following formula as one axiom.[1]

(1) $$p\mathsf{P}q \equiv (p \cdot \sim q)\, \mathsf{P}(\sim p \cdot q)$$

The formula means that a state p is preferred to a state q if and only if p and not q is preferred to not p and q. Hansson pointed out the absurdity of the formula as follows. If $\sim q$ and $\sim p$ are substituted respectively for p and q in (1), we will have the formula:[2]

$$\sim q\, \mathsf{P} \sim p \equiv (\sim q \cdot \sim\sim p)\, \mathsf{P}(\sim\sim q \cdot \sim p),$$

that is

$$\sim q\mathsf{P}\sim p \equiv (p \cdot \sim q)\, \mathsf{P}(\sim p \cdot q).$$

From this formula and (1) we can derive the formula:

(2) $$p\mathsf{P}q \equiv \sim q\mathsf{P}\sim p.$$

He illustrates the invalidity of the formula by the following example. "Suppose that a person A has bought some ticket in a lottery with two prizes of unequal worth . . . . Let p stand for 'A wins the first prize' and q for 'A wins some prize.' It is reasonable to think that $p\mathsf{P}q$ is true for A. If A accepts (2), then he will also claim that $\sim q\mathsf{P}\sim p$ is true, i.e., he will prefer not winning any prize to not winning the first prize."

In this example, one of α and β in $\alpha\mathsf{P}\beta$ logically implies the other. In case where we can think that $\alpha\mathsf{P}\beta$ is equivalent to $\alpha \cdot \sim\beta\mathsf{P}\sim\alpha \cdot \beta$, can we think that $\alpha(\beta)$ logically implies $\beta(\alpha)$? If $\alpha(\beta)$ logically implies $\beta(\alpha)$, $\alpha \cdot \sim\beta(\sim\alpha \cdot \beta)$ is self-contradictory, i.e., it cannot express any logically

1. S. Halldén, *On the Logic of 'Better,'* Lund (1957), p. 28.

2. B. Hansson, "Fundamental axioms for preference relations," *Synthese*, vol. 18 (1968), pp. 428-429.

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