Notre Dame Journal of Formal Logic
Volume XIV, Number 3, July 1973
NDJFAM

DEGREES OF ISOLIC THEORIES

ERIK ELLENTUCK

1. *Introduction.* In this paper* we show that most of the commonly studied isolic structures fall into two categories as far as their first order theories are concerned. They are those whose theory is recursively isomorphic to that of second order arithmetic ($\langle\Lambda, +, \cdot\rangle$ for example) and those whose theory is recursively isomorphic to first order arithmetic ($\langle\Lambda_z, +, \cdot\rangle$ for example). These results are not remarkable, for after all the isols are obtained from $P(\omega)$ by a fairly simple construction. However they do suggest why so many first order questions about Λ reduce to first order questions about ω. And that is because it is hard to find algebraically interesting properties which distinguish Λ from Λ_z. For this reason we believe that it would be quite worth while to continue searching for algebraic distinctions between these structures. Basic concepts concerning Λ and Λ_z are to be found in [5]. The universal theories of Λ and Λ_z have received complete treatments in [11] and [7] respectively, and at least one kind of first order distinction between Λ and Λ_z is contained in [8]. Indeed [8] was the author's chief motivation in undertaking the present study.

We start by defining the isolic structures relevant to our discussion. First of all there are our basic Ω = the recursive equivalence types (commonly called RETs), Λ = the isols, and ω = the non-negative integers. It is also natural to consider $\Lambda(R)$ = the regressive isols (cf. [1]), $\Lambda(H)$ = hyperimmune isols and even $\Lambda(RH) = \Lambda(R) \cap \Lambda(H)$. On the more effective hand we have Ω_f = the RETs of sets with recursively enumerable complement, $\Lambda_z = \Omega_f \cap \Lambda$ = the cosimple isols, $\Lambda_z(R) = \Omega_f \cap \Lambda(R)$ = the cosimple regressive isols, $\Lambda_z(H) = \Omega_f \cap \Lambda(H)$ = the cohypersimple isols, and $\Lambda_z(RH) = \Omega_f \cap \Lambda(RH)$. The latter class has been added for the sake of symmetry only, for by T4 of [3] we have $\Lambda_z(R) = \Lambda_z(RH)$. In order to avoid a cumbersome repetition of names let us introduce a variable W which ranges over the symbols in $\{1, R, H, RH\}$ and use the notation $\Lambda(W)$ or $\Lambda_z(W)$ where

*This paper was prepared while the author was supported by a New Jersey Research Council Faculty Fellowship.

Received February 1, 1971