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THE COMPLETENESS OF COMBINATORY LOGIC WITH DISCRIMINATORS

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1.0 In [2] I introduced a system of combinatory logic with discriminators. Basically this is a system like those presented in [1], modified by the addition of discriminators, or discrimination functions. In this system, the reduction relation $>$ is somewhat different from the reduction relations considered in [1]. The relation $>$ is characterized by transitivity and left monotony—i.e.,

$$(\tau)\ X_1 > X_2,\ X_2 > X_3 \rightarrow X_1 > X_3$$
$$(\nu)\ X_1 > X_2 \rightarrow X_1 Y > X_2 Y.$$

In addition, there is a basic schema for $>$ corresponding to each basic combinator.

1.1 *Pure Combinators*. The pure combinators are the same as those studied in [1]; these are the combinators which do not involve discriminators. The basic pure combinators and their reduction schemata are:

$$\begin{array}{cc} IX > X & WXY > XYY \\ BXY_1Y_2 > X(Y_1Y_2) & SX_1X_2Y > X_1Y(X_2Y) \\ CXY_1Y_2 > XY_2Y_1 & \varphi XX_1X_2Y > X(X_1Y)(X_2Y) \\ KXY > X & \psi X_1X_2Y_1Y_2 > X_1(X_2Y_1)(X_2Y_2) \end{array}$$

Here, as in [1] and [2], parentheses associated to the left are omitted, so that $X_1X_2 \ldots X_n$ is an abbreviation for $(\ldots (X_1X_2) \ldots X_n)$.

It is unnecessary to adopt so many basic pure combinators. For S, K, and C provide a sufficient basis for constructing the rest, as shown below:

$$\begin{array}{ll} I \equiv SKS & \varphi \equiv BBBSB \\ B \equiv S(KS)K & \psi \equiv B\{B[BW(BC)]B\}(BB) \\ W \equiv S(CI) & \end{array}$$

1.2 *Some Definitions*. A *regular* combinator is one whose reduction leaves its first argument unchanged. All of the basic pure combinators are regular. It is sometimes desirable to employ combinators which leave

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