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# FOR SO MANY INDIVIDUALS

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In [2], Tarski introduces the numerical quantifiers. These are expressions $(\exists_k x)$ which mean "there are at least $k$ individuals $x$ such that", where $k$ is any nonnegative integer. Thus $(\exists_1 x)$ is the ordinary quantifier $(\exists x)$. The numerical quantifiers may be defined in terms of the ordinary quantifier and identity as follows:

$$(\exists_0 x)\, A \text{ for } A \rightarrow A$$
$$(\exists_{k+1} x)\, A \text{ for } (\exists_k x)\, (\exists y)\, (-(x = y)\ \&\ A\ \&\ A(y/x)),$$

where $y$ is the first variable which does not occur in $A$ and $A(t/x)$ is the result of substituting a term $t$ for all free occurrences of $x$ in $A$.

Because of their definability, the numerical quantifiers have rarely been considered on their own account. However, in this paper I consider a predicate logic without identity which is enriched with numerical quantifiers as primitive. In section **1**, I present the syntax and semantics for this logic; and in sections **2** and **3**, I establish its completeness.

**1**. *The Logic* L.

*Syntax*

*Formulas* These are constructed in the usual way from relation letters of given degree, (individual) constants, (individual) variables, the truth-functional connectives $\vee$ and $-$, the quantifier $(x)$ and the quantifiers $(\exists_k x)$, $k = 2, 3, \ldots$. We use $(\exists_0 x)\, A$ to abbreviate $A \rightarrow A$ and $(\exists_1 x)\, A$ to abbreviate $(\exists x)\, A$, i.e. $-(x)\, -A$. Also we suppose that there are a denumerable number of individual variables and at least one predicate letter.

*Axioms* (where $k = 2, 3 \ldots$, and $l = 1, 2, \ldots$)

1. *All tautologous formulas*
2. $(x)\, A \rightarrow A(t/x)$, *$t$ free for $x$ in $A$*
3. $(x)\, (A \rightarrow B) \rightarrow ((x)\, A \rightarrow (x) B)$
4. $A \rightarrow (x)\, A$, *$x$ not free in $A$*
5. $(\exists_k x)\, A \rightarrow (\exists_l x)\, A$, $l < k$
6. $(\exists_k x)\, A \leftrightarrow \bigvee_{i=0}^{k} (\exists_i x)\, (A\ \&\ B)\ \&\ (\exists_{k-i} x)\, (A\ \&\ -B)$

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