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ON RELEVANTLY DERIVABLE DISJUNCTIONS

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Where A and B are negation-free formulas of one of the *relevant logics* E, R, or P,[1] we show that

(1) $\vdash A \vee B$ *if and only if* $\vdash A$ *or* $\vdash B$.

Results from [7] and [8] will be presupposed.

1. Our strategy in proving that (1) holds for the relevant logics will be as follows. First, we shall determine a set of conditions such that the negation-free formulas of any logic which simultaneously satisfies all of these conditions has property (1). Second, using results from [7] and [8], we show that the relevant logics satisfy all of these conditions. We close with observations related to the intuitionist logic J and the Lewis system S4, noting now that (1) is one of the more famous properties of J.[2]

2. For present purposes, a logic L is a triple $\langle F, O, T\rangle$, where $\{\rightarrow, \wedge, \vee, -\} = O$, F is a set of formulas built up from sentential variables and the operations of O, and T is the set of theorems of L, which we require to be closed under *modus ponens* for $\rightarrow$, adjunction, and substitution for sentential variables. Where L is $\langle F, O, T\rangle$, an L-theory is any triple $\langle F, O, T'\rangle$, where $T \subseteq T'$ and T' is closed under *modus ponens* for $\rightarrow$ and adjunction. Where no ambiguity results, we identify a theory with its set T' of theorems, and we write $\vdash_{T'} A$ if $A \in T'$.

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1. We assume the sentential logics E and R formulated as in [2] (taking disjunction as an additional primitive). The Anderson-Belnap system P results when Belnap's axioms (1) and (7) are dropped in favor of the weaker scheme $(B \rightarrow C) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$; the implicational part of this system is motivated in Anderson's [1].

2. This fact, and the corresponding fact about S4, may be proved by Gentzen techniques, as e.g. in [3]. (Such techniques have lately been applied by Dunn to secure some of the present results for R; hopefully such techniques will work also for the other relevant logics.) Because of the special character of intuitionist negation, (1) holds without restriction for J, of course; that it holds was first announced in Gödel's [4].

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