

MODAL ELABORATIONS OF PROPOSITIONAL LOGICS

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1. *Modally Augmented Systems* One interesting perspective upon modal logic is obtained by beginning with a nonmodal system, and then developing a modal system "around" it, so to speak, by construing necessity in the "surrounding" modal system as provability within the initial system.¹ Modality, so conceived, is obtained in the broader system by bridging rules linking the necessity operator in this system to thesishood at the nonmodal starting point. The aim of the present paper is to trace out one line of thought along which this idea can be implemented.

Let L be an arbitrary system of (nonmodal) propositional logic based upon negation ($-$), conjunction ($\&$), and implication ($\rightarrow$) as propositional operators. The theses of L are to be derived from certain (at this point unspecified) axioms by the rules of substitution and *modus ponens*. (We shall write $\vdash_X A$ to indicate that A is a thesis of the system X .)

To obtain the modal system ML , the modal augmentation of the initial system L , we introduce the modal operator of necessity ($\Box$) subject to the rules and axioms of the following sort:

I. *Modal Axioms Internal to ML*

(A1) $\vdash_{ML} \Box p \rightarrow p$

(A2) $\vdash_{ML} \Box (p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$

II. *ML -Internal Rules*

(R1) *Substitution*

(R2) *Modus Ponens*

(R3) *Qualified Necessitation: If $\vdash_{ML} A$, then $\vdash_{ML} \Box A$, provided A is not modal-free.*

III. *L/ML Bridging Rule*

(B) *If $\vdash_L A$, then $\vdash_{ML} \Box A$*

1. The origins of this line of thought may be sought in [1]. Here necessity is identified with provability in a certain system. See also [2] and [3].