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RIGHT-DIVISIVE GROUPS

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1 *Introduction* In analogy with the recent developments of the Abelian groups with subtraction as primary operation (Güting [2] and Sobociński [3]) we pose ourselves the question, whether it is possible to develop group theory with right-division as primary operation (a development with left-division as primary operation will be completely analogous).

Right-division will be denoted by /.

$$a/b =_{Df} a \circ b^{-1}.$$

Then the following axioms are necessary (in all axioms the suppositions $a \in \mathfrak{G}$, $b \in \mathfrak{G}$, $c \in \mathfrak{G}$, and $a/b \in \mathfrak{G}$ are silently supposed).

D1 $a/b = a/c \rightarrow b = c$
D2 $(a/((d/d)/b))/c = a/(c/b)$

A set of elements $\mathfrak{D}$ satisfying these axioms is called a *right-divisive group*. It should be proved, that this system of axioms is consistent and independent.

The system is consistent, since it is fulfilled by $D = \{1, -1\}$ with the multiplication as groupoid operation. That *D1* is valid, is trivial. Since $dd = 1$, axiom *D2* takes the form $(a/b)/c = a/(c/b)$, what immediately follows from the commutative and associative laws for the multiplication of integers.

If D is the set of positive integers and the groupoid operation is the addition, axiom *D1* is fulfilled, but axiom *D2* is not, since for $a = b = c = d = 1$

$$(a + ((d + d) + b)) + c = 5 \text{ and } a + (c + b) = 3$$

If D is any set of minimally two elements and $a/b = a$ for all a and b, axiom *D2* is fulfilled, but axiom *D1* is not.

So the axioms are independent.

Theorem 1 *In a right-divisive group* $a/c = b/c \rightarrow a = b$.

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