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LOGICAL AND PROBABILITY ANALYSIS OF SYSTEMS

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1 *Introduction* This paper is concerned with a method for symbolically describing an outcome, event, proposition or system of propositions, premises, assumptions, etc., which is a binary-valued function of n binary elements, and for calculating the probability of an event, based upon Boole's classic, *The Laws of Thought* [1]. We employ concepts and terminology borrowed from fields such as symbolic logic and Boolean algebra, lattice theory and partially ordered sets, which trace their origins to Boole's book. Some new terminology is added, however, for convenience in describing sets. Boole recognized that for a two-valued function of binary elements, the possible outcomes can be algebraically partitioned into two subsets: the "true" or "one" states which are consistent with all of the assumptions or premises of the system, and which we shall call in the sequel the "identity set $\mathfrak{I}$"; and the "false" or "zero" states which are not consistent and which we shall call the "zero set $\mathfrak{O}$." The method described in *Laws of Thought* was to first find the zero set directly from the premises, then subtract it from the universal set to obtain $\mathfrak{I}$. Both $\mathfrak{O}$ and $\mathfrak{I}$ are represented as Boolean polynomials. This is the same thing as building a truth table entirely algebraically, by first finding the false statements and then the true ones, without actually listing out the entire table. Probability assignments and calculations are meaningful only to subsets of $\mathfrak{I}$, because only outcomes included in $\mathfrak{I}$ are consistent with every single characteristic the system is assumed to have.

We follow Boole's method of representing $\mathfrak{O}$ in just about the same way it is done in his book. Each premise or proposition implies that a specified value (either "zero" or "one") for one or more binary elements cannot coexist, under that premise, with a specified value for some other element or elements. Thus, each proposition defines a zero-valued complete subset[1]

1. A complete subset of a partially ordered set contains both its greatest lower bound and its least upper bound, and all elements in the interval between these bounds. An interesting combinatorial result obtained in this paper is that the total number of complete subsets of the universal set is 3^n; this is believed to be new.

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