Notre Dame Journal of Formal Logic
Volume XIX, Number 1, January 1978
NDJFAM

A MODAL SYSTEM PROPERLY INDEPENDENT OF BOTH THE BROUWERIAN SYSTEM AND S4

G. N. GEORGACARAKOS

Although proper subsystems of S5, it is well-known that the Brouwerian system (hereafter referred to as simply 'B') and S4 are independent of each other. This independence, however, is of a peculiar nature: if the proper axiom of either system is appended to the axiomatic basis of the other system, a system deductively equivalent to S5 results. We might say, to coin a new phrase, that these two systems are "properly independent of each other with respect to S5." This rather unusual sense of independence might perhaps lead us to speculate as to whether there exists another system properly independent of both B and S4 with respect to S5; that is, a system such that, if its proper axiom is appended to either the axiomatic basis of B or S4, a system deductively equivalent to S5 results. That there does indeed exist such a system will be shown in section 1. In section 2, we shall examine the modal structure of this system. We shall show that it, like S4, is characterized by possessing exactly fourteen distinct modalities. Finally, in the last section, a Kripke-style semantic interpretation for this system will be offered.

1 An elegant axiomatization of the Classical Propositional Calculus (PC) is afforded by the following three axioms

A1 $CpCqp$
A2 $CCpCqrCCpqCpr$
A3 $CCNpNqCqp$

together with the rules of uniform substitution and detachment. Of course the formation rules and the usual definitions of the other PC connectives are required, but they are familiar enough for them not to be explicitly formulated here. Now if we go on further to append the following two additional axioms

A4 $CLCpqCLpLq$
A5 $CLpp$

Received August 30, 1974