Notre Dame Journal of Formal Logic
Volume XVIII, Number 1, January 1977
NDJFAM

VARIATIONS OF ZORN'S LEMMA, PRINCIPLES OF COFINALITY, AND HAUSDORFF'S MAXIMAL PRINCIPLE. PART II: CLASS FORMS

JUDITH M. HARPER and JEAN E. RUBIN

5 By varying the ordering relation, we obtain a large number of maximal principles. Some are equivalent to the axiom of choice, some are weaker but do not follow from the axioms of set theory, some are provable from the other axioms, and the negations of some are provable from the other axioms. In Part I, [1],* we considered the set forms of maximal principles. In this paper we consider the class or strong forms. The results for class forms are similar to those for sets, but frequently the Axiom of Regularity, **AR**, is used to insure that at various stages of the proofs sets occur and not proper classes.

Section **6** of this paper deals with class forms of Zorn's Lemma and Principles of Cofinality; section **7**, class forms of Hausdorff's maximal principle; and in section **8** we give a list of the statements used in the paper. The notation used is similar to that used in [1]. For convenience, we shall repeat some of the definitions here.

5.1 $\mathbf{NBG}^{\circ}$ denotes von Neumann-Bernays-Gödel set theory excluding the Axiom of Regularity, **AR**, and the Axiom of Choice. $\mathbf{NBG} = \mathbf{NBG}^{\circ} + \mathbf{AR}$. All proofs are in $\mathbf{NBG}^{\circ}$ unless specifically stated otherwise.

5.2 If R partially orders a class X, $y \in X$, and $S = \{u \in X: uRy\}$ then S is called the *R-initial segment of X generated by y* and is denoted by $\overleftarrow{y}$.

5.3 A class X is *ramified* by a relation R iff R partially orders X such that every R-initial segment $\overleftarrow{y}$ of X is linearly ordered by R.

5.4 A class X is a *forest* under the relation R iff R partially orders X such that every R-initial segment $\overleftarrow{y}$ of X is well ordered by R.

*The first part of this paper appeared in *Notre Dame Journal of Formal Logic*, vol. XVII (1976), pp. 565-588.

Received July 24, 1974