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ON THE ELIMINABILITY OF DE RE MODALITIES IN SOME SYSTEMS

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1* M. J. Cresswell has tried to show[1] in [1] that the so called *de re modalities* are not eliminable in the system S, where S = LPC + S5 + **Pr**. The axiom schema **Pr**, or

$$(a)(L\beta a \vee L \sim \beta a) \vee (a)(M\beta a \wedge M \sim \beta a),$$

is deemed by Cresswell to be a fair formal representative of von Wright's *principle of predication.*[2] In this form it is an extremely strong principle. Thus, it can be easily shown that (Lemma 2, section 3)

$$\begin{aligned} \mathrm{LPC} + \mathrm{T} + \mathbf{Pr} \vdash (x_1) \ldots (x_n) \{&L(\alpha x_1 \ldots x_n \equiv \alpha y_1 \ldots y_n) \\ &\vee L(\alpha x_1 \ldots x_n \equiv \sim\alpha y_1 \ldots y_n)\},\end{aligned}$$

where T is the "minimal" modal logic containing the axiom of necessity, the axiom of L-distribution over '$\supset$', and the rule of necessitation.

The above lemma shows, in semantic terms, that **Pr** is strong enough to trivialize modal logic to the extent that the behavior of any context with n free variables is completely determined in any given model by: (a) describing how it behaves "across" the model (i.e., in every "world" therein) for some arbitrary fixed n-tuple and (b) describing how it behaves for each other n-tuple at some world or another. Cresswell's work suggests that this trivialization may not be sufficient to render empty, semantically, the distinction between de re and de dicto modalities.†

Moreover Professor Cresswell suspects[3] that even the further addition of the schema:

$$L(\exists a)\beta \equiv (\exists a)L\beta \qquad \text{(ELC, henceforth)}$$

may not be equal to the job. We shall show by a simple proof, however,

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†(Added in proofs) But meanwhile we have proved in [6] that LPC + S5 + **Pr** $\vdash$ ELC and, therefore, by the present paper's results, that the distinction is rendered empty (even syntactically).

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