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Game-Theoretical Semantics: Insights and Prospects

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*1 **Introductory*** The paradigm problem for game-theoretical semantics (*GTS*) is the treatment of quantifiers, primarily logicians' existential and universal quantifiers. As far as the uses of quantifiers in logic and mathematics are concerned, the basic ideas codified in *GTS* have long been part and parcel of logicians' and mathematicians' folklore. Everyone who has taken a serious course in calculus remembers the definition of what it means for a function $y = f(x)$ to be continuous at x_0: it means that, *given* a number δ however small, *we can find* ϵ such that $|f(x) - f(x_0)| < \delta$ *given* any x such that $|x - x_0| < \epsilon$ (cf. [23], p. 186). The most natural way of making this jargon explicit is to envisage each choice of the value of an existentially bound variable to be my own move in a game and each choice of the value of a universally bound variable a move in the same game by an imaginary opponent. The former is what is covered by such locutions as "we can find", whereas the latter is what is intended by references to what is "given" to us. This is indeed what is involved in the continuity example. For what the above "$\epsilon - \delta$ definition" of continuity says is precisely

(1) $(\delta)(\exists\epsilon)(x)[(|x - x_0| < \epsilon) \supset (|f(x) - f(x_0)| < \delta)]$.

Here "we can find" corresponds to the existential quantifier "$(\exists\epsilon)$" and the locution "given" to the universal quantifiers "(δ)" and "(x)". Game-theoretical treatment of the two quantifiers is to all intents and purposes just a systematization of the ideas involved in this example.

Logicians have even introduced a name for the functions that embody my strategy in choosing values for existentially bound variables. These are what is meant by *Skolem functions* in logic. Using $s(z)$ as such a function, we can for instance express what (1) says by asserting that a suitable Skolem function exists for my choice of ϵ, i.e., asserting that

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