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Every Quotient Algebra for C_1 is Trivial

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1 In recent years, a number of different types of logics have been proposed with the intention of avoiding the various paradoxes of material implication, particularly the property that from a contradiction anything may be deduced. Two such types of logics are the relevance logics of Anderson and Belnap [1], and the paraconsistent logics in the vicinity of C_1. The logic C_1 has primitives $\neg$, $\supset$, &, $\vee$, and is given axiomatically below. In the opinion of this author, C_1 has various unsatisfactory features, two of which are that it lacks the theorem $A \supset \neg\neg A$, and that the rule of replacement ($\vdash A \equiv B$ implies $\vdash C(A) \equiv C(B)$, for any context C; $A \equiv B$ being defined as usual by $(A \supset B)$ & $(B \supset A)$) does not hold for C_1.

To date, there has been an outstanding problem (raised, for example, in [10], p. 508) about C_1: how to "algebraise" it. The aim of this paper is to contribute to the solution of that problem by proving that on certain very minimal assumptions C_1 has no nontrivial quotient algebra. We will say presently what it means for a quotient algebra to be trivial. It is suggested that the present result, in addition to "solving" the algebraisation problem, exhibits a further unsatisfactory feature of C_1, namely that C_1 lacks a proper biconditional. We hope to amplify this point in a later paper.

The present enterprise is to investigate the consequences of partitioning the formula algebra of C_1 into a quotient algebra of equivalence classes by some relation $\sim$ holding between formulas. The relation $\sim$ need *not* necessarily be syntactic, i.e., definable by a formula in the operators $\neg$, $\supset$, &, $\vee$. We impose the following four requirements on any such relation $\sim$ and quotient algebra: (a) $\sim$ is an equivalence relation, i.e., $A \sim A$, $A \sim B$ implies $B \sim A$, and $A \sim B$ and $B \sim C$ imply $A \sim C$. (b) The formula algebra is homomorphic to the quotient algebra (with corresponding operations) obtained from the equivalence relation; i.e., $A \sim B$ implies $C(A) \sim C(B)$, for any context C. (c) If $A \sim B$ and $\vdash A$ then $\vdash B$ (where '$\vdash$' means provability in C_1). This is necessary to prevent including

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