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DEDUCTIVE COMPLETENESS AND CONDITIONALIZATION IN SYSTEMS OF WEAK IMPLICATION

M. RICHARD DIAZ, Jr.

I wish to investigate the conditions under which certain systems of implication satisfy deductive completeness of the kind associated with the deduction theorem (in the sense of Curry and Feys [2]). The systems that I investigate have received considerable attention in the last two decades: the implication fragment of Relevance Logic, **R**→ (Church's Weak Implication); the implication fragment of strict implication, **S4**→; the implication fragment of Anderson and Belnap's System of Entailment, **E**→; and the system of Ticket Entailment, **T**→. None of these systems satisfy deductive completeness except under certain conditions which may be interpreted as the satisfaction of conditions of relevance (for **R**→), modality (for **S4**→), relevance and modality (for **E**→), and inference ticket/inference distinctions (**T**→). Thus we might say that they are each deductively complete for an extended notion of deductive completeness.

In Section 2, I formulate natural deduction systems **NR**→, **NS4**→, **NE**→, and **NT**→ which are deductively equivalent respectively to **R**→, **S4**→, **E**→, and **T**→. Each system involves only two rules, one of which is *modus ponens* and one a form of conditionalization. The conditionalization rule in each case is based on the deduction theorem of the corresponding axiom system. Furthermore, each system is the result of adding a further restriction to only the rule of conditionalization for the previous system. In this form we can see more clearly the relationship between the systems and intuitionistic implication (**H**→); and what relevance, necessity, and ticket entailment amount to. Finally, in Section 3, I show how to formulate **T**→ in terms of a restriction on the rule for *modus ponens*, and how adding this restriction to *modus ponens* in **E**→, **R**→, **S4**→, and **H**→ affects these systems.

Let **S** be a deductive system with an implication operator, $\supset$, which satisfies the rule *modus ponens* (MP): $A, A \supset B \vdash_{\mathbf{S}} B$. Curry and Feys [2] called such a system deductively complete if, whenever from a premise B, and possibly other premises, we can derive A, then we can derive $B \supset A$ from these other premises alone. It has been shown by Gentzen [3] that

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