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A SIMPLE GENERALIZATION OF TURING COMPUTABILITY

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1 *Introduction* In the present paper,* the notion of a Turing machine is generalized as follows: the notion of an *ideal computation* is defined to be a finite sequence of expressions of some arbitrary language. An *ideal machine* is then defined to be a set of *ideal computations* such that functionality from first to last elements of the *ideal computations* in the *ideal machine* obtains. Then, using the devices of Gödel numbering, a *length* function (applied to *ideal computations*) and *input* and *output* functions (which, when applied to *ideal computations*, select their first and last elements, respectively), a generalized analogue of the Kleene T-predicate is defined. A class of *ideal machines* is then said to be an $\mathbf{R}$-class just in case it is Gödel numerable and contains an *ideal machine* which, in a sense made precise, *decides* the generalized T-predicate analogue.

The following results will be obtained: (1) The classes of Turing machine computations and Post normal system proofs are $\mathbf{R}$-classes. (2) There exist $\mathbf{R}$-classes which properly include the class of Turing machines. (3) For any $\mathbf{M}$, where $\mathbf{M}$ is an $\mathbf{R}$-class, there exist functions which are not computable by $\mathbf{M}$. This theorem generalizes the Turing unsolvability of the "halting problem", and its proof makes use of the familiar diagonal procedure. Defining $\mathbf{M}$-decidability for $\mathbf{R}$-classes $\mathbf{M}$ in the natural way, we have that: (4) In general, the class of $\mathbf{M}$-decidable sets is not closed under Boolean operations on pairs of sets. (5) There exist trivial $\mathbf{R}$-classes, indeed $\mathbf{R}$-classes which contain only one *ideal machine*. (6) In general, $\mathbf{R}$-classes are not invariant with respect to Gödel numbering. (7) Mostowski's generalization of recursive function theory, $\mathfrak{R}$-definability (developed in [1]), is properly included in the notion of $\mathbf{R}$-ness.

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